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AUTOMATION AND REMOTE CONTROL

Volume 19, Number 5 May, 1958

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SAMPLED-DATA SYSTEMS WITH EXTRAPOLATING DEVICES*

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(Moscow)

Automatic sampled-data systems containing extrapolating devices are investigated. Equations for such systems, describing the processes at any moment of time, are given. The method of analyzing such systems is illustrated by examples.

INTRODUCTION

Recent years have been characterized by the widespread use of discrete methods of measuring, transmitting and transforming physical quantities. Discrete methods of measurement give the possibility of measuring physical quantities without using devices the states of which are characterized by these quantities, and also allow the measurement of those quantities whose definition involves the passage of time.

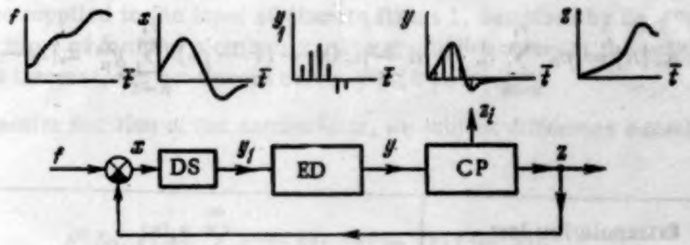

Fig. 1.

Discrete methods of transmitting and transforming are distinguished by their high accuracy and imperviousness to noise, and allow one and the same channel to be used for the transmission of a large number of data. These methods are used in automatic sampled-data systems and in automatic systems with digital computers.

In many cases, the use of such systems gives rise to the necessity of reproducing a continuous function from data transmitted only at equally-spaced intervals of time, T_p. This problem leads to the reconstruction, in the time interval $nT_p < t < (n+1)T_p$, of the continuous function from its values at the previous moments of time, $t = mT_p$, where $m = n-1, n-2, \ldots, n-k$, that is, to the problem of extrapolation.

Since discrete data generally contain random components, it is necessary, in addition to extrapolation, to smooth or adjust the discrete data.

The investigation of sampled-data systems, designed to transform discrete data to continuous data, was expounded in works [1-3]. In this paper, we consider automatic sampled-data systems in which an extrapolating device (ED),

* This paper contains one section of the talk, "Certain questions in the theory of discrete systems," presented at the Symposium on the Applications of Computers in Automatic Systems, held at Atlantic City in October, 1957.

i. e., a transformer of discrete data to continuous data, is one of the system elements. A block diagram of such a system is given in Fig. 1. The figure also gives the character of the variation of the output of each system element. The darkened segments show the computation of the output z of the continuous portion of the system from the input quantity f.

The output of the data-sampling (DS) element (discrete data) is fed to the input of ED, which transforms it into a continuous quantity which changes, between sampling times, according to a definite law. Generally, this law is taken as a polynomial of degree s. Such an ED will henceforth be called an s-order ED, or transformer. A zero-order ED corresponds to a fixed element, which retains a discrete value until the appearance of the next discrete value. A first-order ED is implemented by a linear extrapolator, etc. The output of the ED acts on the continuous portion of the system (CP).

Since the extrapolation under consideration is intimately related to the known character of the processes during the repitition periods, i. e., between the discrete sampling times, the equations for a system with an extrapolating device must describe the processes at any moment of time.

1. Structure and Equations of the Extrapolating Device

We begin by clarifying the structure of an ED.

If the law for transforming the discrete data to continuous data is restricted to be a polynomial of degree s, then the output of an s-order ED may be represented by the equation

$$y[n, \varepsilon] = \sum_{\nu=0}^{s} \alpha_\nu [n] \varepsilon^\nu, \tag{1}$$

where $0 \leq \epsilon \leq 1$ is the relative time measured from the beginning of each interval t = n, and the coefficients $\alpha_\nu [n]$ change from interval to interval. In the general case, the coefficients satisfy the difference equation

$$\alpha_\nu [n] = p_0 \sum_{\mu=0}^{r_1} \xi_\mu^{(\nu)} y_1 [n-\mu, 0] - (1-p_0) \sum_{\mu=1}^{r_1} g_\mu^{(\nu)} \alpha_\nu [n-\mu]. \tag{2}$$

TABLE 1

Formula no.	Extrapolation law	$\alpha_\nu[n]$
1	Taylor's Series	$\alpha_\nu [n] = \frac{y^{(\nu)} [n]}{\nu!} \approx \frac{\Delta^\nu y_1 [n-\nu, 0]}{\nu!}$
2	Newton's Formula	$\alpha_\nu [n] = \sum_{\mu=\nu}^{n} (-1)^{\mu-1} S_{\mu-1}^{\mu} \frac{\Delta^\mu y_1 [n-\mu, 0]}{\mu!}$, where $S_{\mu-1}^{\mu}$ — Sterling number
3	Method of least squares	$\alpha_0 [n] = \frac{2}{k(k+1)} \sum_{\mu=1}^{k} (k-3\mu+1) y_1 [n-\nu, 0]$,
	(linear extrapolation)	$\alpha_1 [n] = \frac{6}{k(k^2-1)} \sum_{\mu=1}^{k} (k-2\mu+1) y_1 [n-\nu, 0]$

Here p_0 is a constant multiplier, $\xi_\mu^{(\nu)}$, $g_\mu^{(\nu)}$ are constants, $y_1[n, 0]$ is the input and $\alpha_\nu [n]$ is the output. For $p_0 = 0$ or $p_0 = 1$, the magnitude of $\alpha_\nu [n]$ depends only on, respectively, the output or input quantity. For $0 < p_0 < 1$, the magnitude of $\alpha_\nu [n]$ depends both on the input and the output, and the degree of this dependency is determined by the magnitude of p_0.

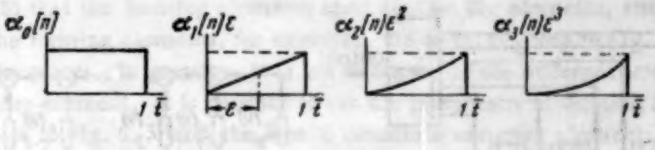

Fig. 2.

The law of variation of the coefficients $\alpha_\nu[n]$ depends on the choice of the extrapolation law. Different methods of extrapolation are quite well described in the specialized mathematical literature (cf., for example, [4]). Certain particular cases of laws of variation of the coefficients $\alpha_\nu[n]$ are given in Table 1.

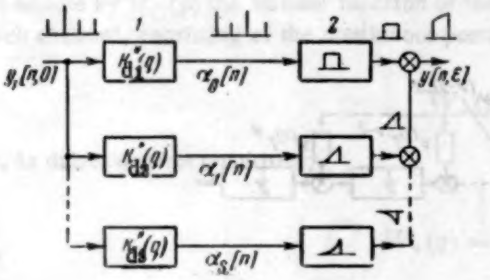

Fig. 3.

From Formulas (1) and (2), it is possible to determine the structure of the ED.

Each term $\alpha_\nu[n]\,\epsilon^\nu$ in (1), during the interval when ϵ varies from 0 to 1, determines a "graduated pulse" (Fig. 2), i. e., a pulse of parabolic shape, with height equal to $\alpha_\nu[n]$, which varies from interval to interval according to the law defined by difference equation (2).

The graduated pulses can be formed by supplying instantaneous "square" pulses, $\alpha_\nu[n]$, to an element whose transfer function equals the representation of the graduated pulse [5].

Thus, the structure of an ED will have the form shown in Fig. 3.

The discrete data are supplied to the input of discrete filters 1, described by Eq. (2). The filter output, also discrete, is fed to the input of forming elements 2. The graduated pulses at the output of these elements, when added, form, in each interval, the continuous curve $y[n, \epsilon] = y(\bar{t})$.

To determine the transfer function of the extrapolator, we subject difference equation (2) to the discrete Laplace transform [6, 7]:

$$F^*(q, \varepsilon) = \sum_{n=0}^{\infty} e^{-qn} f[n, \varepsilon] = D\{f[n, \varepsilon]\},$$

where $\epsilon = 0$.

Using the notation $A_\nu^*(q) = D\{\alpha_\nu[n]\}$, $Y^*(q, 0) = D\{y[n, 0]\}$, we obtain the equation of the discrete filter with respect to the representation:

$$A_\nu^*(q) = K_{d\nu}^*(q)\, Y^*(q, 0), \tag{3}$$

where

$$K_{d\nu}^*(q) = \frac{p_0 \sum_{\mu=0}^{r_1} \xi_\mu^{(\nu)} e^{-q\mu}}{1 + (1 - p_0) \sum_{\mu=1}^{r} g_\mu^{(\nu)} e^{-q\mu}} \tag{4}$$

is the transfer function of the discrete filter which, for $p_0 < 1$, may be considered as a sampled-data system with feedback.

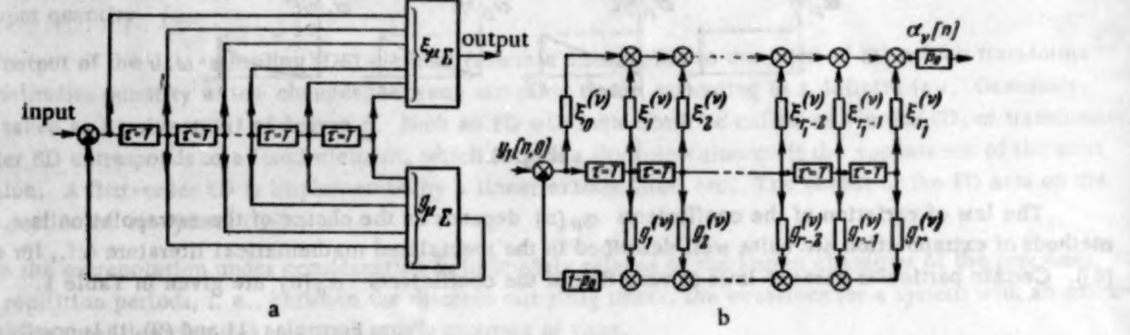

Fig. 4.

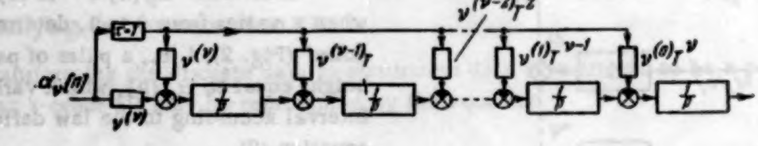

Fig. 5.

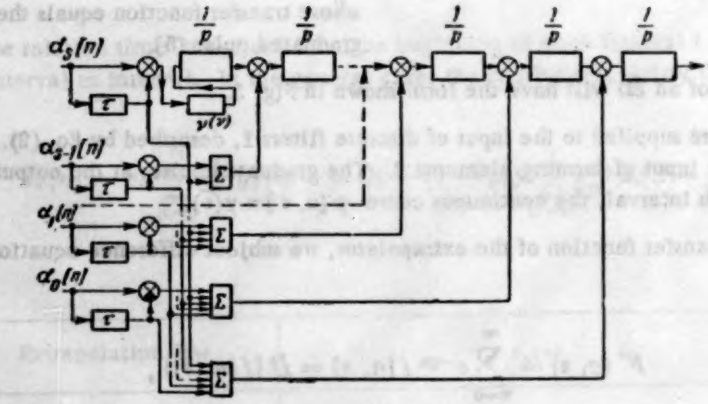

Fig. 6.

A discrete filter may be realized in the form of a circuit with lag elements and amplifiers (Fig. 4a). A more detailed layout of the circuit is given in Fig. 4b [6].

It is also possible to use, as a discrete filter, a digital computer (DC), the program for which is determined by difference equation (2) or by transfer function (4) [6].

By definition, the transfer function of the forming elements must coincide with the transform of the graduated pulses, i. e.,

$$K_{F_\nu}(p) = \int_0^{T_p} t^\nu e^{-pt}\, dt = \frac{\nu^{(\nu)}}{p^{\nu+1}} - e^{-pT_p} \sum_{\mu=0}^{\nu} \frac{\nu^{(\nu-\mu)}}{p^{\nu-\mu+1}} T_p^{\mu}, \tag{5}$$

where

$$\nu^{(\nu-\mu)} = \nu(\nu-1)\ldots(\mu+1) = \frac{\nu!}{\mu!} \tag{6}$$

is a factorial polynomial.

It follows from (5) that the forming elements must contain lag elements, amplifiers and integrators. The schematic for one of the forming elements, for example, the ν-th, is given in Fig. 5. With increasing ν, the number of integrators increases. In practice, it is not necessary to use different integrators for each ν-th (ν = = 0, 1, 2, . . ., s) forming element. It is possible to use the integrators of the s-th forming element for all forming elements, as is shown in Fig. 6. Here, the sign Σ denotes a summing element.

Thus, an ED consists of two parts, a discrete part (discrete filters) and a continuous part (integrators) [2].

2. Equation of a Closed Sampled-Data System with an Extrapolating Device

A schematic for a closed sampled-data system with an ED may take the form given in Fig. 7. In the total continuous portion of the system (total CP) are now included in the integrators of the forming elements (FE). If we denote by $W_C(p)$ the transfer function of the continuous portion of the system, then the transfer function of each channel, consisting of the continuous portion and of a forming element, will equal

$$W_\nu(p) = K_{F\nu}(p)\, W_C(p)$$

or, in dimensionless quantities,

$$W_\nu(q) = \frac{1}{T_p} K_{F\nu}\left(\frac{q}{T_p}\right) W_C\left(\frac{q}{T_p}\right), \tag{7}$$

where $q = pT_p$.

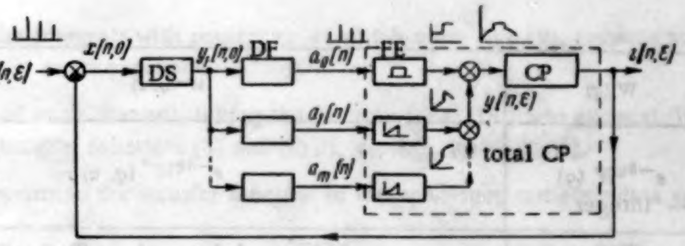

Fig. 7.

This relationship follows from the similarity theorem, according to which if $W_\nu(p) = L\{w_\nu(t)\}$, then $L\{w_\nu(T_p \bar{t})\} = \frac{1}{T_p} W_\nu\left(\frac{q}{T_p}\right)$, where $\bar{t} = \frac{t}{T_p}$ and $q = pT_p$.

We substitute into (7) the value for $K_{F\nu}(p)$, first replacing p in it by $\frac{q}{T_p}$. We get the result

$$W_\nu(q) = T_p^{\nu+1}\left\{\frac{\nu^{(\nu)}}{q^{\nu+1}} - e^{-q}\sum_{\mu=0}^{\nu}\frac{\nu^{(\nu-\mu)}}{q^{\nu-\mu+1}}\right\} W_C(q). \tag{8}$$

Here, we took $W_C(q) = \frac{1}{T_p} W_C\left(\frac{q}{T_p}\right)$

For determining $W_\nu^*(q)$, it is convenient to use Tables 2 and 3, which establish the relationships between $W(q)$ and $W^*(q)$. The derivations of these relationships may be found in, for example, [7, 9, 11].

We shall assume initially that $W_C(q)$ is given in analytic form. On the basis of Formula(3)in Table 2, we can also find $W_C^*(q, \epsilon)$ in analytic form. With the use of Expressions 1 and 5 of Table 3, we can write the transfer function $W_\nu^*(q, \epsilon)$ corresponding to (8), in the form

TABLE 2

Formula no.	$W^*(q, \varepsilon)$	$W(q)$	Remarks
1	$W^*(q, \varepsilon) = \sum_{n=0}^{\infty} e^{-qn} w[n, \varepsilon]$	$w[n, \varepsilon] = \frac{1}{2\pi j} \int_{\sigma-j\pi}^{\sigma+j\pi} W^*(q, \varepsilon) e^{qn} dq$	$w[n, \varepsilon] = w(n+\varepsilon)$ Impulse characteristic of the continuous pattern $w(t)$ for $\bar{t} = n + \varepsilon$
2	$W^*(q, \varepsilon) = \sum_{r=-\infty}^{\infty} e^{(q+2\pi jr)\varepsilon} W(q + 2\pi jr)$	$W(q) = \int_0^1 W^*(q, \varepsilon) e^{-q\varepsilon} d\varepsilon$	$W(q) = \frac{1}{T} W\left(\frac{q}{T}\right)$ Transfer function of the continuous portion
3	$W^*(q, \varepsilon) = \sum_{\nu=0}^{s} \sum_{\mu=0}^{r_\nu - 1} \frac{C_{\mu\nu}}{\mu!} \times \frac{d^\mu}{dq_\nu^\mu} \left(\frac{{}^r e^q}{e^q - e^{q_\nu}} e^{q_\nu} \right)$ Here $C_{\nu\mu} = \frac{1}{(r_\nu - \mu - 1)!} \lim_{q \to q_\nu} \frac{d^{r_\nu - \mu - 1}}{d_{q_\nu} r_\nu - \mu - 1} [W(q)(q - q_\nu)^{r_\nu}]$	$W(q) = \sum_{\nu=0}^{s} \sum_{\mu=0}^{r_\nu - 1} \frac{C_{\nu\mu}}{\mu!} \frac{d^\mu}{dq_\nu^\mu} \left(\frac{1}{q - q_\nu} \right)$	Poles $W(q)$. $q_0 = 0$ multiple r_0, $q_\nu \neq 0$ multiple r_ν $(\nu = 1, 2 \ldots s)$

TABLE 3

Formula no.	$W(q)$	$W^*(q, \varepsilon)$
1	$e^{-kq} W(q)$ k — integer	$e^{-kq} W^*(q, \varepsilon)$
2	$e^{-\gamma q} W(q)$ $(\gamma < 1)$	$e^{-q} W^*(q, 1 + \varepsilon - \gamma) \quad (0 \leqslant \varepsilon \leqslant \gamma)$, $W^*(q, \varepsilon - \gamma) \quad (\gamma \leqslant \varepsilon \leqslant 1)$
3	$qW(q)$	$\frac{\partial W^*(q, \varepsilon)}{\partial \varepsilon}$
4	$q^\nu W(q)$ $\lim q^\nu W(q) = 0$	$\frac{\partial^\nu W^*(q, \varepsilon)}{\partial \varepsilon^\nu}$
5	$\frac{1}{q} W(q)$	$\int_0^\varepsilon W^*(q, \varepsilon) d\varepsilon + \frac{1}{e^q - 1} \int_0^1 W(q, \varepsilon) d\varepsilon$
6	$\frac{1}{q^\nu} W(q)$	$\underbrace{\int_0^\varepsilon \ldots \int_0^\varepsilon}_{\nu \text{ times}} W^*(q, \varepsilon) d\varepsilon^\nu + \frac{1}{e^q - 1} \sum_{\eta=0}^{\nu-1} \frac{\varepsilon^{\nu-\eta-1}}{(\nu - \eta - 1)} \int_0^1 W^*_{(\eta)}(q, \varepsilon) d\varepsilon$, where $W^*_{(\eta)}(q, \varepsilon) = \int_0^\varepsilon W^*_{(\eta-1)}(q, \varepsilon) d\varepsilon + \frac{1}{e^q - 1} \int_0^1 W^*_{(\eta-1)}(q, \varepsilon) d\varepsilon$ and $W^*_0(q, \varepsilon) = W^*(q, \varepsilon)$

$$W^{*}_{\nu}(q,\varepsilon)=T_{\mathrm{p}}^{\nu+1}\Bigg\{\gamma^{(\nu)}\underbrace{\int_{0}^{\varepsilon}\ldots\int_{0}^{\varepsilon}}_{\nu+1\ \text{times}}W^{*}_{\mathrm{C}}(q,\varepsilon)\,d\varepsilon^{\nu+1}-$$

$$-e^{-q}\sum_{\mu=0}^{\nu}\gamma^{(\nu-\mu)}\underbrace{\int_{0}^{\varepsilon}\ldots\int_{0}^{\varepsilon}}_{\nu-\mu+1\ \text{times}}W^{*}_{\mathrm{C}}(q,\varepsilon)\,d\varepsilon^{\nu-\mu+1}+$$

$$+\frac{\gamma^{(\nu)}}{e^{q}-1}\sum_{\eta=0}^{\nu}\frac{\varepsilon^{\nu-\eta}}{(\nu-\eta)!}\int_{0}^{1}W^{*}_{(\eta-1)}(q,\varepsilon)\,d\varepsilon-$$

$$-\frac{e^{-q}}{e^{q}-1}\sum_{\eta=0}^{\nu-\mu}\frac{\varepsilon^{\nu-\mu-\eta}}{(\nu-\mu-\eta)!}\int_{0}^{1}W^{*}_{(\eta-\mu-1)}(q,\varepsilon)\,d\varepsilon\Bigg\}, \tag{9}$$

where

$$W^{*}_{(\eta)}(q,\varepsilon)=\int_{0}^{\varepsilon}W^{*}_{(\eta-1)}(q,\varepsilon)\,d\varepsilon+\frac{1}{e^{q}-1}\int_{0}^{1}W^{*}_{(\eta-1)}(q,\varepsilon)\,d\varepsilon$$

$$W^{*}_{(0)}(q,\varepsilon)=W^{*}_{\mathrm{C}}(q,\varepsilon).$$

The computation of the integrals with respect to ϵ, which enter into (9), presents no difficulty, since ϵ enters either as a multiplier or as an exponent.

The transfer function of each channel, taking the discrete filter (DF) into account, is found by the simple cross-multiplication of the transfer functions (4) and (9), i. e., $K^{*}_{\mathrm{d}\nu}(q)\,W^{*}_{\nu}(q,\epsilon)$.

We can now easily determine the transfer function of the open-loop sampled-data system with an ED:

$$W^{*}(q,\varepsilon)=\sum_{\nu=0}^{s}K^{*}_{\mathrm{d}\nu}(q)\,W^{*}_{\nu}(q,\varepsilon). \tag{10}$$

The equation for the closed-loop sampled-data system with an ED is written according to a formula known from the theory of discontinuous regulation

$$Z^{*}(q,\varepsilon)=\frac{W^{*}(q,\epsilon)}{1+W^{*}(q,\epsilon)}F^{*}(q,\varepsilon)$$

in the form

$$Z^{*}(q,\varepsilon)=\frac{\sum\limits_{\nu=0}^{s}K^{*}_{\mathrm{d}\nu}(q)\,W^{*}_{\nu}(q,\epsilon)}{1+\sum\limits_{\nu=0}^{s}K^{*}_{\mathrm{d}\nu}(q)\,W^{*}_{\nu}(q,\epsilon)}F^{*}(q,\varepsilon). \tag{11}$$

$$K^*_C(q, \varepsilon) = \frac{\sum\limits_{\nu=0}^{s} K^*_{d\nu}(q)\, W^*_\nu(q, \varepsilon)}{1 + \sum\limits_{\nu=0}^{s} K^*_{d\nu}(q)\, W^*_\nu(q, \varepsilon)} \tag{12}$$

is the transfer function of the closed-loop sampled-data system with an ED. If we replace q in (12) by $j\overline{\omega} = j\omega T_p$, we obtain the frequency characteristic of the closed-loop sampled-data system.

Particular cases subsumed under Eq. (12) are the equation for a sampled-data system with an indexer (for $s = 0$) [3, 5], and the equation of a system for transforming discrete data into continuous data (for $W_C(q) = 1$) [2].

3. Different Forms of Presenting the Transfer Functions of Open-Looped Systems

In those cases when the transfer function of the invariable continuous portion $W_C(q)$ is unknown or complicated, it is possible to use experimentally obtained impulse or frequency characteristics to determine the transfer function $W^*(q, \epsilon)$, or the frequency characteristic $W^*(j\overline{\omega}, \epsilon)$.

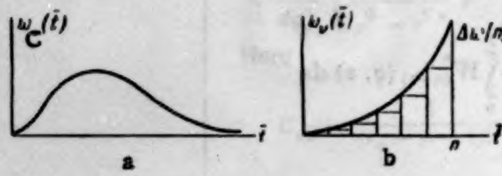

Fig. 8.

Let the impulse characteristic of the continuous portion $w_C(\overline{t})$ have the form represented in Fig. 8a.

As follows from (8), the impulse characteristic for each channel, exclusive of the discrete filters, is written in the form

$$w_\nu(\overline{t}) = T_p^{\nu+1}\left\{\gamma^{(\nu)} \underbrace{\int_0^{\overline{t}} \cdots \int_0^{\overline{t}}}_{\nu+1 \text{ times}} w_C(\overline{t})\, d\overline{t}^{(\nu+1)} - \sum_{\mu=0}^{\nu} \gamma^{(\nu-\mu)} \underbrace{\int_1^{\overline{t}} \cdots \int_1^{\overline{t}}}_{\nu-\mu+1 \text{ times}} w_C(\overline{t})\, d\overline{t}^{\nu-\mu+1}\right\}. \tag{13}$$

Integrating graphically ν times ($\nu = 1, 2, \ldots, s$) the impulse characteristic , and making use of Formula (13), we find $w_\nu(\overline{t})$ (Fig. 8b). Since $w_\nu(\overline{t})$ grows with increasing $\overline{t}$, the following formula should be used to determine $W^*_\nu(q, \epsilon)$

$$W^*_\nu(q, \varepsilon) = \frac{1}{(e^q - 1)^k} \sum_{n=0}^{\infty} e^{-qn} \Delta^k w_\nu[n, \varepsilon] + \frac{e^q}{e^q - 1} \sum_{\nu=1}^{k-1} \frac{\Delta^k w_\nu[0, \varepsilon]}{(e^q - 1)^\nu}, \tag{14}$$

which follows from Formula (1) of Table 2 if the theorem on the transform of differences [7] is used. There may always be found a k such that

$$\lim_{n \to \infty} \Delta^k w_\nu[n, \varepsilon] = 0, \tag{15}$$

and in practice, (14) may be limited to a finite number of terms.* The expression thus found for $W^*_\nu(q, \epsilon)$ are substituted into Eq. (11).

* A similar formula, for $k = 1$ and $q = j\overline{\omega}$, was used in [8].

In the case where it is difficult to find an analytic expression for $W_\nu^*(q, \epsilon)$, given the frequency characteristic of the continuous portion $W_C(j\bar{\omega})$, it is easy to find the frequency characteristics $W_\nu^*(j\bar{\omega}, \epsilon)$, and thereafter to use the frequency method [9] to compute the sampled-data system.

According to Formula(2)of Table 2 we have, for $q = j\bar{\omega}$,

$$W_\nu^*(j\bar{\omega}, \varepsilon) = \sum_{r=-\infty}^{\infty} e^{j(\bar{\omega}+2\pi r)\varepsilon} W_\nu[j(\bar{\omega}+2\pi r)] =$$

$$= \sum_{r=-\infty}^{\infty} e^{j(\bar{\omega}+2\pi r)\varepsilon} \overline{K}_{d\nu}[j(\bar{\omega}+2\pi r)] W_C[j(\bar{\omega}+2\pi r)], \tag{16}$$

where $\overline{K}_{F\nu}(j\bar{\omega}) = K_{F\nu}\left(\frac{j\bar{\omega}}{T_p}\right)$.

Substituting $W_\nu^*(j\bar{\omega}, \epsilon)$ into (10) for $q = j\bar{\omega}$, and changing the order of summation, we obtain, taking periodicity into account,

$$K_{d\nu}^*(j\bar{\omega}) = K_{d\nu}^*[j(\bar{\omega}+2\pi r)], \tag{17}$$

with the expression for $W^*(j\bar{\omega}, \epsilon)$ in the form

$$W^*(j\bar{\omega}, \varepsilon) = \sum_{r=-\infty}^{\infty} e^{j(\bar{\omega}+2\pi r)\varepsilon} W_C[j(\bar{\omega}+2\pi r)] \times$$

$$\times \left\{\sum_{\nu=0}^{s} K_{d\nu}^*[j(\bar{\omega}+2\pi r)] \overline{K}_{F\nu}[j(\bar{\omega}+2\pi r)]\right\}. \tag{18}$$

As a consequency of Formula (18) we obtain the following method for constructing the frequency characteristic of an open-looped sampled-data system with an ED.

Changing the numerical frequency characteristic $W_C(j\omega)$ by the relationship $\bar{\omega} = \omega T_p$, and multiplying its modulus by $1/T_p$, we obtain $W_C(j\bar{\omega})$. Rotating the radius-vector of $W_C(j\bar{\omega})$ counterclockwise through an angle $\bar{\omega}\epsilon$ (ϵ = const), we find

$$e^{j\bar{\omega}\varepsilon} W_C(j\bar{\omega}). \tag{19}$$

Then we construct the curve

$$\sum_{\nu=0}^{s} K_{d\nu}^*(i\bar{\omega}) \overline{K}_{F\nu}(j\bar{\omega}). \tag{20}$$

Cross-multiplying curves (19) and (20), we find

$$e^{j\bar{\omega}\varepsilon} W_C(j\bar{\omega}) \sum_{\nu=0}^{s} K_{d\nu}^*(j\bar{\omega}) \overline{K}_{F\nu}(j\bar{\omega}). \tag{21}$$

Using well-known methods [9], we obtain $W^*(j\bar{\omega}, \epsilon)$ from this curve in accordance with Formula (18).

4. Determination of the Processes in Closed-Loop Systems with Extrapolating Devices

Methods known from the theory of discontinuous regulation may be applied to Eq. (12) for the investigation of sampled-data systems.

If $W^*(q, \epsilon)$ is found in analytic form, $K_c^*(q, \epsilon)$ may be given in the form

$$K_c^*(q, \varepsilon) = \frac{H^*(q, \varepsilon)}{G^*(q)}, \tag{22}$$

where $H^*(q, \epsilon)$ and $G^*(q)$ are polynomials in e^q.

The impulse characteristic of the closed-loop system, $k_c[n, \epsilon]$, is found from the decomposition formula [7]

$$k_c[n, \varepsilon] = \sum_{\nu=0}^{l} \frac{H^*(\bar{q}_\nu, \varepsilon)}{\dot{G}(\bar{q}_\nu)} e^{\bar{q}_\nu (n-1)}. \tag{23}$$

Here $\bar{q}_\nu$ are the poles of $K_c^*(q, \epsilon)$ which, for simplicity, we shall assume are simple poles.

If $W^*(j\bar{\omega}, \epsilon)$ is found graphically then, by means of the method described in [9], it is possible to find the frequency characteristic of the closed-loop system, or its real part $\operatorname{Re} K_c^*(j\bar{\omega}, \epsilon)$. Then, from the formula

$$k_c[n, \varepsilon] = \frac{2}{\pi} \int_0^\pi \operatorname{Re} K_c^*(j\bar{\omega}, \varepsilon) \cos \bar{\omega} n \, d\bar{\omega} \tag{24}$$

it is possible to determine the impulse characteristic of the closed-loop sampled-data system.

The process for an arbitrary impulse $f[n, \epsilon]$ is found from the superposition formula

$$z[n, \varepsilon] = \sum_{m=0}^{n} k_c[m, \varepsilon] f[n-m, 0]. \tag{25}$$

5. Example of the Computation of Systems with Extrapolating Devices

As an example, we consider an automatic system with a linear ED, i. e., with a first-order transformer. The block schematic of this system is shown in Fig. 1.* We shall assume that only the frequency characteristic of the continuous portion of the system without the extrapolator is known (Fig. 9).

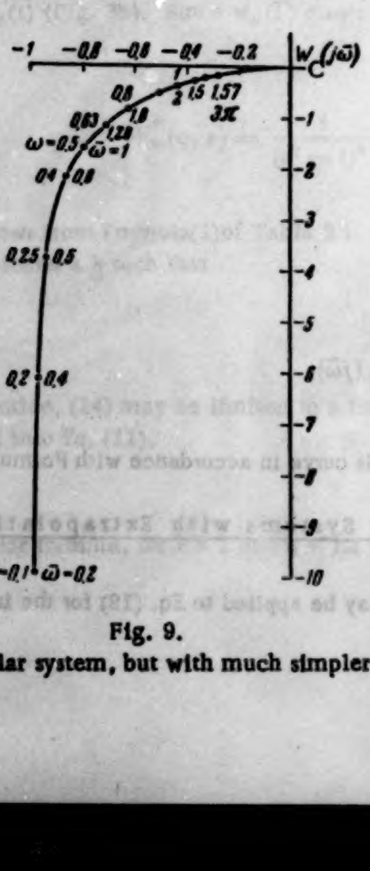

Fig. 9.

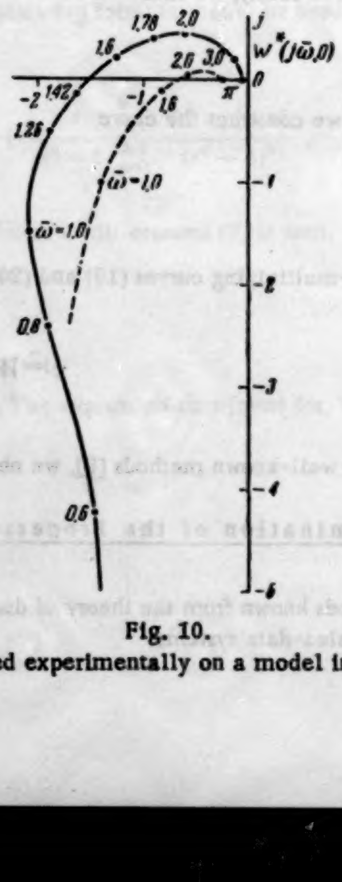

Fig. 10.

* A similar system, but with much simpler assumptions, was studied experimentally on a model in [10].

The extrapolation law is determined by Eq. (1) for s = 1, where

$$\alpha_0[n] = \frac{1}{6}(5y_1[n,0] + 2y_1[n-1.0] - y_1[n-2.0]),$$

$$\alpha_1[n] = \frac{1}{2}(y_1[n,0] - y_1[n-2.0]).$$

These values are found by the method of least squares from the condition that the best straight line be passed through the previous three points (Formula(3)of Table 1 for k = 3).

According to Formula (4), the frequency characteristics of the discrete filters will be

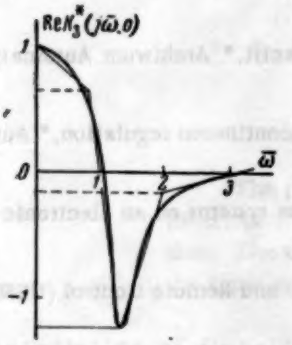

Fig. 11.

$$K^*_{d0}(j\bar{\omega}) = \frac{1}{6}(5 + 2e^{-j\bar{\omega}} - e^{-j2\bar{\omega}}),$$
$$K^*_{d1}(j\bar{\omega}) = \frac{1}{2}(1 - e^{-j2\bar{\omega}}). \tag{26}$$

The frequency characteristics of the forming links, according to Formula (5), with $p = \frac{j\bar{\omega}}{T_p}$, will have the form:

$$\bar{K}^*_{F0}(j\bar{\omega}) = T_p \frac{1 - e^{-j\bar{\omega}}}{j\bar{\omega}},$$
$$\bar{K}^*_{F1}(j\bar{\omega}) = -\frac{T_p^2}{\bar{\omega}^2}(1 - e^{-j\bar{\omega}} - j\bar{\omega}e^{-j\bar{\omega}}). \tag{27}$$

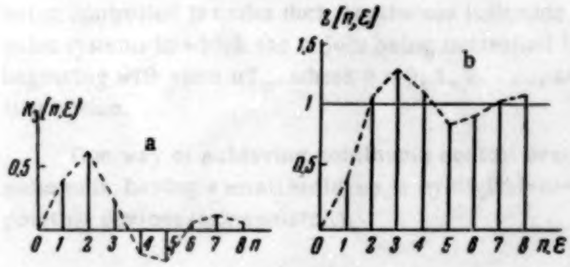

Fig. 12.

On the plane of the frequency characteristic $W_C(j\bar{\omega})$ we construct the curve from the expression $K^*_{d0}(j\bar{\omega})\bar{K}^*_{F0}(j\bar{\omega}) + K^*_{d1}(j\omega)\bar{K}^*_{F1}(j\bar{\omega})$. Multiplying this by $W_C(j\bar{\omega})$ and extending this periodically, we find $W^*(j\bar{\omega}, 0)$. Figure 10 shows the curve for $W^*(j\bar{\omega}, 0)$ for T_p = 2 sec and k_0 = 1, where k_0 is the total gain; the dotted curve gives the frequency characteristic for the open-looped system with a zero-order extrapolator ($\alpha_1[n] = 0$).

As is clear from a comparison of these characteristics, the use of a linear extrapolator lowers the stability of the system. From $W^*(j\bar{\omega}, 0)$, using ordinary graphic methods [9], the real part of the frequency characteristic of the closed-loop system $\operatorname{Re} K^*_C(j\bar{\omega}, 0)$ is constructed. This is shown in Fig. 11, where the open-loop gain is taken to be $k_0 = {}^1/_3$. By approximating $\operatorname{Re} K^*_C(j\bar{\omega}, 0)$ by the sum of elementary trapezoids, we find the impulse characteristic of the closed-loop system $k_C[n, \epsilon]$ as the sum of the impulse characteristics corresponding to the individual trapezoids (Fig. 12a). The process for jump impulses is determined according to Formula (24) using simple summation, and has the form shown in Fig. 12b.

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A WAY OF FORMING TRANSFER FUNCTIONS OF SAMPLED-DATA CONTROL SYSTEMS WITH EXTRAPOLATING DEVICES

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The paper deals with a way of forming conditional transfer functions of extrapolating devices for digital-to-analog conversion. The transfer function expression depends on the shape of the input pulses.

1. Characteristics of Considered Control System

There exist control systems which contain a section (pulse unit) which converts a continuous function of time into a series of its values equally spaced in time, and which are characterized by the fact that the object being controlled is under their continuous influence. This last condition distinguishes these systems from pure pulse systems in which the object being controlled is acted upon periodically, during certain intervals of time, beginning with time nT_p, where $n = 0, 1, 2, \ldots$, and during the rest of the time is not subjected to any controlling action.

One way of achieving continuous control over an object, with systems containing the above-mentioned pulse unit, having a small build-up, is by digital-to-analog (or sectionally-continuous) conversion using extrapolating devices (extrapolators).

The present work sets forth a method of forming the transfer functions of extrapolating devices, which permits the application of the existing theory of sampled-data control to the analysis and synthesis of systems containing these devices.

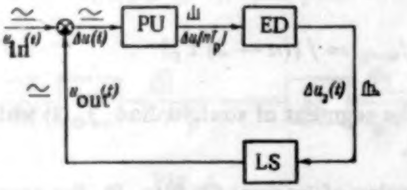

Fig. 1.

Figure 1 shows a block schematic of the type of control system being considered, where PU is the pulse unit, ED is the extrapolating device, LS is the linear (continuous) section of the system, including the object being controlled.

Discrete values $\Delta u\,[nT_p]$ of the continuous function

$$\Delta u\,(t) = u_{in}(t) - u_{out}\,(t) \tag{1}$$

permit the construction of the sectionally-continuous (or continuous) function $\Delta u_e(t)$, similar in a specific sense to the function $\Delta u(t)$, which we are considering, and thus fill in the gap in our knowledge of the two functions being compared, $u_{in}(t)$ and $u_{out}(t)$. This operation is carried out by the extrapolating device ED, which thus predicts the nature of the change in the function $\Delta u(t)$ between the times nT_p when the measurement is made.

Let us state one of the possible methods of extrapolation.

In the intervals

$$nT_p \leqslant t < (n+1)\,T_p \tag{2}$$

let us draw sections of straight lines (Fig. 2) in such a way that the sum of the squares of the inclinations of each straight line from the k of the preceding points is a minimum.

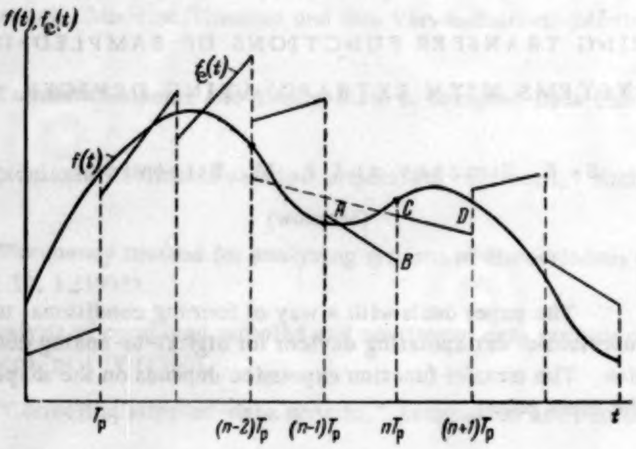

Fig. 2.

The equations of the indicated straight lines can be easily derived in a general form, but we shall limit ourselves to the simplest example of linear extrapolation from three preceding measurements. The method presented below can be applied to more complicated cases.

Direct computations, which result in the finding of the minimum of a function with several variables, lead, in the case of extrapolation from three preceding measurements, to the following expression:

$$f_e(t) = \frac{t - nT_p}{2T_p}\,(f_n - f_{n-2}) + \frac{1}{6}\,(5f_n + 2f_{n-1} - f_{n-2}). \tag{3}$$

In Eq. (3)

$$f_n = f\,[nT_p], \quad f_{n-1} = f\,[(n-1)\,T_p], \quad f_{n-2} = f\,[(n-2)\,T_p] \tag{4}$$

are the measured values of the function $f(t) = \Delta u(t)$, from which the segment of straight line $f_e(t)$ which predicts the behavior of the function $f(t)$ on Interval (2) is constructed.

Let us call the magnitude $f_e\,[nT_p]$, corresponding to the beginning of Interval (2) (Fig. 2), the average value of the function, and the magnitude $f_e\,[(n+1)\,T_p]$, corresponding to the end of Interval (2), the predicted value. The introduced notation permits one to distinguish between the beginning and end of each piece of the sectionally-continuous function (3). The discontinuities in the extrapolated function $f_e(t)$, constructed in accordance with (3), have an undesirable effect on the dynamics of the control system. These discontinuities can be avoided by changing the method of extrapolation.

One of the possible methods of obtaining a continuous extrapolated function $f_e(t)$ is shown in Fig. 3. Here the position of the predicted point B is determined from the three preceding measurements: $f\,[(n-1)\,T_p]$, $f\,[(n-2)\,T_p]$ and $f\,[(n-3)\,T_p]$. At time nT_p the value $f\,[nT_p]$ is fed to the extrapolator. The presence of this value together with the preceding values $f\,[(n-1)\,T_p]$ and $f\,[(n-2)\,T_p]$ permits, at this same time nT_p, the determination of the position of the following predicted point D. It is therefore possible to move in the direction BD while avoiding the jump from section AB to section CD. The equation of the straight-line section BD is determined, in our example, from the four preceding values of the input variable $f(t)$.

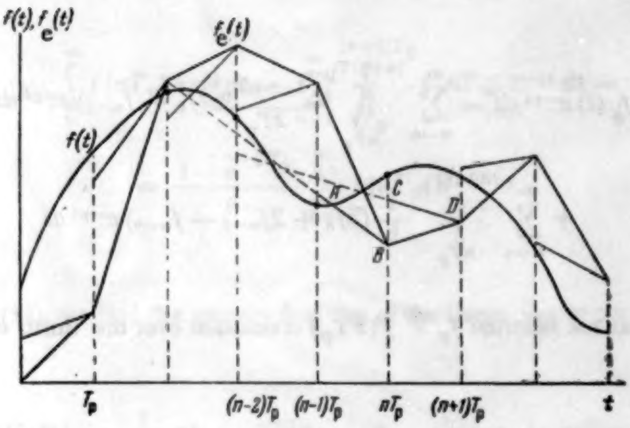

Fig. 3.

It follows from what has been said that the graph of the output variable from the extrapolator with the continuous output (Fig. 3) can be obtained from the consecutive connection of the predicted points of all the intervals. The corresponding analytical expressions are tabulated in the Appendix.

If the extrapolating device is looked upon as a section of the control system, then the extrapolating equations tabulated in the Appendix (among them Eq. (3)), establish the dependence of the output variable $f_e(t)$ on the input $f(t)$. This dependence is expressed by means of a differential equation.

The other characteristic of the extrapolator is the fact that the output $f_e(t)$ is determined only by the values $f[\nu T_p]$ ($\nu = 0, 1, 2, \ldots$) and does not depend on the form of the function $f(t)$ between the points of measurement, νT_p. It is possible to construct many functions $f(t)$, coinciding at the times νT_p, which correspond to one and the same function $f_e(t)$ at the output of the extrapolator. It is, therefore, not possible to form the transfer function of the extrapolating device (as a relationship between the input and output) using methods based on the theory of continuous control.

2. Derivation of the Transfer Function of the Extrapolator

The extrapolator can be looked upon as consisting of the pulse unit PU and the linear part with the transfer function $W_e(p)$ (Fig. 4). The pulse unit converts a continuous function $f(t)$ into a multistep one $f[\nu T_p]$. In accordance with the theory of sampled-data control [1], such a unit is characterized by an amplification factor $k_I = 1$ build-up $\gamma = 1$.

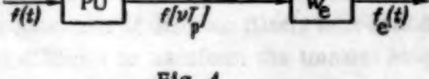

Fig. 4.

We shall consider the multistep function $f[\nu T_p]$ to be the input of the section, representing the linear portion of the extrapolator, and the function $f_e(t)$ to be the output of this section. Equation (3) expresses the relationship between them.

Taking the Laplace transform of both sides of Eq. (3):

$$\int_0^\infty f_e(t)\, e^{-pt}\, dt = \int_0^\infty \frac{t - nT_p}{2T_p} (f_n - f_{n-2})\, e^{-pt}\, dt + \\ + \int_0^\infty \frac{1}{6} (5f_n + 2f_{n-1} - f_{n-2})\, e^{-pt}\, dt. \qquad (5)$$

In view of the fact that the integrand on the right side of (5) changes from interval to interval, we shall break up the integrals in (5) into a sum of integrals:

$$\int_0^\infty f_e(t)\,e^{-pt}\,dt=\sum_{n=0}^{\infty}\int_{nT_p}^{(n+1)T_p}\frac{t-nT_p}{2T_p}(f_n-f_{n-2})\,e^{-pt}\,dt+$$

$$+\sum_{n=0}^{\infty}\int_{nT_p}^{(n+1)T_p}\frac{1}{6}(5f_n+2f_{n-1}-f_{n-2})\,e^{-pt}\,dt. \qquad (6)$$

Utilizing the fact that the function $f_\nu = f[\nu T_p]$ is constant over the limits of every integral in (2), we transform (6) to:

$$\int_0^\infty f_e(t)\,e^{-pt}\,dt=\sum_{n=0}^{\infty}\frac{1}{2T_p}(f_n-f_{n-2})\int_{nT_p}^{(n+1)T_p}(t-nT_p)\,e^{-pt}\,dt+$$

$$+\sum_{n=0}^{\infty}\frac{1}{6}(5f_n+2f_{n-1}-f_{n-2})\int_{nT_p}^{(n+1)T_p}e^{-pt}\,dt. \qquad (7)$$

Simple calculations, with due attention to the designations in (4) yield:

$$\int_0^\infty f_e(t)\,e^{-pt}\,dt=\frac{1}{2p^2T_p}\,(1-e^{-pT_p}-pT_pe^{-pT_p})\times$$

$$\times\sum_{n=0}^{\infty}\{f[nT_p]-f[(n-2)T_p]\}\,e^{-pnT_p}+\frac{1}{6p}\,(1-e^{-pT_p})\sum_{n=0}^{\infty}\times$$

$$\times\{5f[nT_p]+2f[(n-1)T_p]-f[(n-2)T_p]\}\,e^{-pnT_p}. \qquad (8)$$

Assuming, as usual, that $f(t)\equiv 0$ for $t<0$, it can be shown that

$$\sum_{n=0}^{\infty}f[(n-\alpha)T_p]\,e^{-pnT_p}=e^{-\alpha pT_p}\sum_{n=0}^{\infty}f[nT_p]\,e^{-pnT_p}. \qquad (9)$$

Equation (9) permits the simplification of Expression (8):

$$\int_0^\infty f_e(t)\,e^{-pt}\,dt=\Big[\frac{1}{2pT_p}(1-e^{-pT_p}-pT_pe^{-pT_p})(1+e^{-pT_p})+$$

$$+\frac{1}{6}\,(5+2e^{-pT_p}-e^{-2pT_p})\Big]\frac{1-e^{-pT_p}}{p}\sum_{n=0}^{\infty}f[nT_p]\,e^{-pT_p}. \qquad (10)$$

On the other hand, we find what must be the expression for the input variable $f[nT_p]$:

$$\int_0^\infty f[nT_p]\, e^{-pt}\, dt = \sum_{n=0}^{\infty} \int_{nT_p}^{(n+1)T_p} f[nT_p]\, e^{-pt}\, dt =$$

$$= \frac{1-e^{-pT_p}}{p} \sum_{n=0}^{\infty} f[nT_p]\, e^{-pnT_p}. \tag{11}$$

Comparing (10) and (11), we find the transfer function of the linear part of the extrapolator:

$$W_e(p) = \frac{1}{2pT_p}(1-e^{-pT_p}-pT_p e^{-pT_p})(1+e^{-pT_p}) + \\ + \frac{1}{6}(5+2e^{-pT_p}-e^{-2pT_p}), \tag{12}$$

as a relationship between the expressions for the output and input values.

Let us call Operator (12) the conditional transfer function. Further investigations of the system are carried out following well-known rules of sampled-data control.

The question of the formation of transfer functions of extrapolating devices has been also considered using a different approach, by Ia. Z. Tsypkin [3]. The author represents the extrapolator by a complex pulse unit which forms pulses of a definite shape (see [2]).

It can be shown that the pulse transfer function [4]

$$W_e^*(q) = \sum_{m=-\infty}^{\infty} W_e(q+2\pi mi)\, \frac{1-e^{-(q+2\pi mi)}}{q+2\pi mi} \tag{13}$$

(where $q = pT_p$), built up by the method of conditional transfer functions, is identical to the corresponding transfer function of the extrapolator derived by Ia. Z. Tsypkin [3].

For this we must make use of Relationship (8), given in [2], and take into account the periodicity of the transfer functions of discrete filters introduced, in accordance with [3], into the circuit of the extrapolator. Then it is not difficult to transform the transfer function, derived by Ia. Z. Tsypkin, into the form of (13). It is also possible to achieve the reverse transformation into a conditional transfer function.

Calculations have shown that the use of the indicated transfer function is convenient in those cases where it is necessary to examine the effect of different methods of extrapolation on the operation of control systems. The application of conditional transfer functions shortens calculations in cases where one is interested in the choice of the optimum parameters of the linear part of the system without changing, at the same time, the constitution of the extrapolator.

SUMMARY

In the study of the dynamics of a control system containing a series-connected pulse unit, having infinitely small buildup ($\gamma \longrightarrow 0$), and an extrapolating device, these sections can be replaced by a pulse unit forming rectangular pulses having buildup $\gamma = 1$, and the linear (continuous) part of the extrapolating device, also connected in series.

The transfer function of the linear part of the extrapolating device can be found by means of the usual (not discrete) Laplace transforms. This transfer function expresses the connection between the expressions for the input and output functions only for a particular form of input – a series of rectangular pulses having buildup $\gamma = 1$. Hence such a function may be called a conditionsl transfer function.

The above material allows us to assume that the method of conditional transfer functions (in the indicated sense) is applicable when the input consists of a series of pulses of arbitrary, known form.

APPENDIX

The table lists formulas relating the output variable $f_e(\bar{t})$ of the linear extrapolating device to the discrete values of the input variable $f(\bar{t})$, as well as the corresponding conditional transfer functions $W_e(q)$.

The time $\bar{t} = t/T_p$ is expressed in terms of the interval time T_p. Hence, in particular, $q = pT_p$.

Method of extrapolation	Extrapolation formula for Interval (2) $(n = 0, 1, 2, \ldots)$	Conditional transfer function $W_e(q)$
ES-1	$f_e(\bar{t}) = f[n]$	1
ES-2	$f_e(\bar{t}) = (\bar{t} - n)\{f[n] - f[n-2]\} + f[n]$	$\frac{1}{q}(1 - e^{-q})(1 + q)$
ES-3	$f_e(\bar{t}) = \frac{1}{2}(\bar{t} - n)\{f[n] - f[n-2]\} + \frac{1}{6}\{5f[n] + 2f[n-1] - f[n-2]\}$	$\frac{1}{2q}(1 + e^{-q}) \times (1 - e^{-q} - qe^{-q}) + \frac{1}{6}(5 + 2e^{-q} - e^{-2q})$
ES-4	$f_e(\bar{t}) = \frac{1}{10}(t - n)\{3f[n] + f[n-1] - f[n-2] - 3f[n-3]\} + \frac{1}{10}\{7f[n] + 4f[n-1] + f[n-2] - 2f[n-3]\}$	$\frac{1}{10q}(3 + 4e^{-q} - 3e^{-2q}) \times (1 - e^{-q} - qe^{-q}) + \frac{1}{10}(7 + 4e^{-q} + e^{-2q} - 2e^{-3q})$
ES-5	$f_e(\bar{t}) = \frac{1}{10}(\bar{t} - n)\{2f[n] + f[n-1] - f[n-3] - 2f[n-4]\} + \frac{1}{5}\{3f[n] + 2f[n-1] + f[n-2] - f[n-4]\}$	$\frac{1}{10q}(2 + 3e^{-q} + 3e^{-2q} + 2e^{-3q})(1 - e^{-q} - qe^{-q}) + \frac{1}{5}(3 + 2e^{-q} + 2e^{-2q} - e^{-4q})$
EC-2	$f_e(\bar{t}) = (\bar{t} - n)\{f[n] - f[n-1]\} + f[n-1]$	$\frac{1}{q}(1 - e^{-q})$
EC-3	$f_e(\bar{t}) = (\bar{t} - n)\{2f[n] - 3f[n-1] + f[n-2]\} + 2f[n-1] - f[n-2]$	$\frac{1}{q}(1 - e^{-q})(2 - e^{-q})$
EC-4	$f_e(\bar{t}) = \frac{1}{3}(\bar{t} - n)\{4f[n] - 3f[n-1] - 3f[n-2] + 2f[n-3]\} + \frac{1}{3}\{4f[n-1] + f[n-2] - 2f[n+3]\}$	$\frac{1}{3q}(4 + e^{-q} - 2e^{-2q}) \times (1 - e^{-q} - qe^{-q}) + \frac{1}{3}(4e^{-q} + e^{-2q} - e^{-3q})$

Method of extrapolation	Extrapolation formula for Interval (2) (n = 0, 1, 2, . . .)	Conditional transfer function $W_e(q)$
EC-5	$f_e(\bar{t}) = \frac{1}{2}(\bar{t}-n)\{2f[n] - f[n-1] - f[n-2] - f[n-3] + f[n-4]\} + \frac{1}{2}\{2f[n-1] + f[n-2] - f[n-4]\}$	$\frac{1}{2q}(2 + e^{-q} - e^{-3q}) \times (1 - e^{-q} - qe^{-q}) + \frac{1}{2}(2e^{-q} + 2e^{-3q} - e^{-4q})$
EC-6	$f_e(\bar{t}) = \frac{1}{10}(\bar{t}-n)\{8f[n] - 3f[n-1] - 3f[n-2] - 3f[n-3] - 3f[n-4] + 4f[n-5]\} + \frac{1}{10}\{8f[n-1] + 5f[n-2] + 2f[n-3] - f[n-4]\,4f - [n-5]\}$	$\frac{1}{10q}(8 + 5e^{-q} + 2e^{-2q} - e^{-3q} - 4e^{-4q}) \times (1 - e^{-q} - qe^{-q}) + \frac{1}{10}(8e^{-q} + 5e^{-2q} + 2e^{-3q} - e^{-4q} - 4e^{-5q})$

The following designations are introduced in the table: ES-3 is an extrapolator with a sectionally-continuous output function $f_e(\bar{t})$, determined from the three preceding values of $f[\nu]$ (ν = n, n − 1, n − 2) of the input function $f(\bar{t})$ (see Fig. 2); EC-4 is an extrapolator with a continuous output function $f_e(\bar{t})$, determined from the four preceding values $f[\nu]$ (ν = n, n − 1, n − 2, n − 3) of the input function $f(\bar{t})$ (see Fig. 3), etc.

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AN ANALYTICAL METHOD OF SYNTHESIS OF LINEAR CONTROL SYSTEMS IN THE PRESENCE OF INTERFERENCE AND WITH A SPECIFIED DYNAMIC ACCURACY

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An analytical method of approximation of transcendental transfer functions of automatic control systems is proposed. The functions were obtained with the help of fraction-rational functions [10, 14]. The practical applications of the method are illustrated by several examples.

INTRODUCTION

The raising of the requirements of modern technology for speed, dynamic accuracy, and quality of automatic control systems, providing separation of signal from noise, brings out the necessity for the development of an engineering method for the calculation of these systems, with a view to obtaining optimum designs and an accurate evaluation of their working characteristics. This is particularly important in the fields of radar tracking systems and automatic control systems. In a general case there appears at the input of the indicated systems a useful signal

$$\beta_i(t) = s(t) + \sum_{k=1}^{n} a_k f_k(t), \tag{1}$$

composed of a random signal s(t) and a series of known functions of time $f_1(t), \ldots, f_n(t)$, as well as a random noise signal $\beta_n(t) = n(t)$. Moreover, s(t) and n(t) may be stationary or nonstationary random signals, while the coefficients $a_1, \ldots, a_n$ may be arbitrary, unknown quantities or random variables with a given mutual correlation

$$E[a_i a_j] = \gamma_{ij}, \quad i,j = 1, \ldots, n. \tag{2}$$

The theory of the interpolation and extrapolation of stationary random sequences was originally developed by A. N. Kolmogorov [1]. Subsequently, the problem of the optimization of a linear, dynamic system for the case when the input consists solely of continuous stationary random functions s(t) and n(t), was solved in [2].

It was shown in [3-6] that for automatic control systems it is often more expedient to examine and establish the statistical properties of the derivatives of the useful signal s(t). The resulting optimum solutions, obtained in this case, are related to a class of linear, astatic systems which have practical applications. The statistical theory of optimization, developed by Wiener, was generalized in [7] for the case when the signal and noise are both nonstationary random functions. The optimum pulse transfer function of the system $w(t,\tau)$, for a nonstationary input, represents a linear system with variable parameters, its response being determined by the superposition integral

$$\beta_0(t) = \int_0^{\infty} w(t, \tau)[\beta_i(\tau) + \beta_n(\tau)]\, d\tau. \tag{3}$$

A method of constructing an optimum system, with constant (in time) parameters, which ensured, in a specific sense, the filtering of nonstationary random functions, was proposed in [8, 9]. The theory of the optimization of linear dynamic systems was further advanced in paper [10], for the case when the observation time T is finite, s(t) and n(t) are the stationary, random functions, and the mathematical expectation of the function s(t) is unknown, but is determined on the interval T by a function of time in the form of a polynomial of degree r. Davis [11] generalized the problem, formulated in [10], for the case of nonstationary random functions s(t) and n(t). V. S. Pugachev [12] derived the general, necessary and sufficient condition for a minimum in the mean square error of a dynamic system, this condition being the source of all the known equations of optimum, dynamic systems studied in the above categories. Among the shortcomings of papers [10, 11] is, firstly, the uselessness of the final equations, from the point of view of their physical realizability [13] for practical calculations. An attempt was made in [14] to develop a method for determining the optimum pulse transfer function, stated in [10], for its application in the calculation of linear tracking systems. By means of some deviations from optimum conditions, concrete, physically realizable solutions were obtained in [14]. These deviations consisted of the following: the error coefficients were given beforehand as end values, as a result of which the reproduction error, specified by the regular useful signal, was not equal to zero; a graphical approximation to the optimum, transcendental, transfer function of the system was introduced by means of the required fraction-rational function of the minimum phase type. Another attempt was made in [15], where the optimum solutions for the case of the slowly varying (in time) regular signal, were obtained with the help of the minimum, practical, limiting error of reproduction criterion. This paper solves the problem of how, starting with the optimum, but usually nonrealizable characteristics, obtained in [10], one can determine characteristics which are physically realizable and which have the least deviation from the optimum ones.

1. Statement of the Problem

Let a useful signal (1) act on the input of the automatic control system in question, during which s(t) appears as a stationary, random function having a known autocorrelation function $R_{ss}(\tau)$ or spectral density $G_{ss}(\omega)$. We assume, moreover, that the function of time $\sum_{k=1}^{n} a_k f_k(t)$ is given, and is chosen on the basis of the most unfavorable or typical operating conditions of the system. The noise n(t) occurs as a stationary, random function with a given autocorrelation function $R_{nn}(\tau)$ or spectral density $G_{nn}(\omega)$, and we assume for the sake of simplicity that the functions s(t) and n(t) are not mutually correlated.

Designating the pulse transfer function of the linear, automatic control system by $w(\tau)$, we can write the response of the system to the input $\beta_i(t) + \beta_n(t)$ in the form

$$\beta_0(t) = \int_0^{T} w(\tau)[\beta_i(t-\tau) + \beta_n(t-\tau)]\, d\tau, \tag{4}$$

where $w(\tau) = 0$ when $0 \geq \tau \geq T$.

Let us designate by $w_I(\tau)$ the ideal pulse transfer function of the automatic control system (ACS). Then, in the absence of noise, the response of the system to the input signal $\beta_i(t)$ will have the form

$$\beta_{0I}(t) = \int_{-\infty}^{\infty} w_I(\tau)\beta_i(t-\tau)\, d\tau. \tag{5}$$

The problem is reduced to the finding of a function $w(\tau)$ for which minimum dispersion occurs in the control errors:

$$\overline{\delta_\beta^2}(t) = E[\{\beta_0(t) - \beta_{0l}(t)\}^2]. \tag{6}$$

The total control error will be of the form

$$\delta_\beta(t) = \int_0^T w(\tau)[s(t-\tau) + n(t-\tau)]\,d\tau - \int_{-\infty}^{\infty} w_l(\tau)\,s(t-\tau)\,d\tau + \\ + \sum_{k=1}^{n} a_k \left[\int_0^T w(\tau) f_k(t-\tau)\,d\tau - \int_{-\infty}^{\infty} w_l(\tau) f_k(t-\tau)\,d\tau\right]. \tag{7}$$

The last summand in Eq. (7) determines the systematic control error. In [10] the requirement for the systematic error to be equal to zero is introduced, which, on the basis of (7), will be expressed mathematically in the form

$$\int_{-\infty}^{\infty} w_l(\tau) f_k(t-\tau)\,d\tau = \int_0^T w(\tau) f_k(t-\tau)\,d\tau, \quad k = 1, \ldots, n. \tag{8}$$

Following [14] we shall suppose that the systematic control error $\delta_{\beta_s}(t)$ is not equal to zero, and, therefore, Eq. (8) is not satisfied. In this case we can write the following set of equations:

$$\begin{aligned} &\int_{-\infty}^{\infty} w_l(\tau) f_1(t-\tau)\,d\tau - \int_0^T w(\tau) f_1(t-\tau)\,d\tau = \Delta_1(t), \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &\int_{-\infty}^{\infty} w_l(\tau) f_k(t-\tau)\,d\tau - \int_0^T w(\tau) f_k(t-\tau)\,d\tau = \Delta_k(t), \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &\int_{-\infty}^{\infty} w_l(\tau) f_n(t-\tau)\,d\tau - \int_0^T w(\tau) f_n(t-\tau)\,d\tau = \Delta_n(t), \end{aligned} \tag{9}$$

whence the systematic error is

$$\delta_{\beta_s}(t) = \sum_{k=1}^{n} a_k \Delta_k(t). \tag{10}$$

Because of the technical requirements which are imposed on the control system, the maximum permissible values of the functions $\Delta_k(t)$ ($k = 1, \ldots, n$) can be often chosen in such a way that the inequality $|\delta_{\beta_s}|_{max} \leq \epsilon$ is satisfied, where ϵ is the maximum permissible value of the systematic control error. Subsequently, the problem is reduced to finding, for the given functions $R_{ss}(\tau)$, $R_{nn}(\tau)$, $G_{ss}(\omega)$, $G_{nn}(\omega)$ and the observation time T, the pulse transfer function $w(\tau)$ which satisfies the set of equations (9) and which guarantees a minimum in the dispersion of the random error:

$$\delta_{\beta_{ra}}(t) = \int_0^T w(\tau)[s(t-\tau) + n(t-\tau)]\,d\tau - \int_{-\infty}^{\infty} w_l(\tau)\,s(t-\tau)\,d\tau. \tag{11}$$

For simplicity we shall assume that the mathematical expectation of the functions $s(t)$ and $n(t)$ are identically equal to zero.

2. Determination of the Optimum Control Characteristics

The solution of the variation on the problem in [10] formulated above reduces to the necessity for the following integral equation to be satisfied:

$$\int_0^T w(\tau)\left[R_{ss}(t-\tau)+R_{nn}(t-\tau)\right]d\tau=\sum_{k=0}^{n}\lambda_k t^k+$$

$$+\int_{-\infty}^{\infty} w_1(\tau)R_{ss}(t-\tau)\,d\tau,\quad (0\leqslant t\leqslant T), \tag{12}$$

where λ_k is a Lagrange multiplier.

The solution of Eq. (12) has the following form:

$$w(t)=\sum_{k=0}^{n}A_k t^k+\sum_{k=1}^{2m}B_k e^{\alpha_k t}+\frac{1}{2\pi}R(p)\times$$

$$\times\int_{-\infty}^{\infty}\frac{G_{ss}(\omega^2)}{A(\omega^2)}W_1(j\omega)R(-j\omega)e^{j\omega t}d\omega.+ \tag{13}$$

$$+C_1\delta(t)+\cdots+C_{l-m}\delta^{(l-m-1)}(t)+D_1\delta(t-T)+\cdots+D_{l-m}\delta^{(l-m-1)}(t-T)$$
$$(0<t<T),$$
$$w(t)=0\quad(0\geqslant t\geqslant T),$$

where $A_0, A_1, \ldots, A_n$, $B_1, B_2, \ldots, B_{2m}$, $C_1, C_2, \ldots, C_{l-m}$, $D_1, D_2, \ldots, D_{l-m}$ are constants to be determined, $\delta(t)$ is the delta function, $\delta^{(\nu)}(t)$ is the νth derivative of the delta function.

$$G_{ss}(\omega^2)+G_{nn}(\omega^2)=\frac{A(\omega^2)}{B(\omega^2)}=|\Psi(j\omega)|^2, \tag{14}$$

$2m$ is the power of the major term of $A(\omega^2)$, $2l$ is the power of the major term of $B(\omega^2)$, $p\equiv d/dt$, $W_1(p)=$ $=\int_0^{\infty} w_1(t)e^{-pt}dt$ is the ideal control operator and

$$\Psi(p)=\frac{Q(p)}{R(p)}=\frac{\gamma_0+\gamma_1 p+\cdots\gamma_m p^m}{\eta_0+\eta_1 p+\cdots+\eta_l p^l}. \tag{15}$$

The polynomials $Q(p)$ and $R(p)$ do not have any zeros in the right half-plane of the variable p.

Substituting $w(t)$ from (13) into (12) and treating the resulting expression as an identity, we obtain $2l$ linear, homogeneous equations, which contain the unknown constants A_k, B_k, C_k, and D_k. Substituting $w(t)$ from (13) into (9) with $k = 0, 1, \ldots, n$, we obtain another set of $n + 1$ eqs. The solution of the obtained $2l + n + 1$ equations yields the values of all the constant coefficients.

The optimum transfer function of the equation of a closed system is, as is well known, of the following form:

$$K(p)=\int_0^{\infty} w(t)\,e^{-pt}dt. \tag{16}$$

Substituting the expression for the pulse transfer function w(t) in (13) into (16) we obtain

$$\begin{aligned}K(p)&=\int_0^T\sum_{k=0}^{n}A_kt^ke^{-pt}dt+\sum_{k=1}^{2m}\frac{B_k}{p+\alpha_k}-e^{-pT}\sum_{k=1}^{2m}\frac{B_ke^{\alpha_kT}}{p+\alpha_k}+\\&+\frac{1}{2\pi}R(p)\int_0^T e^{-pt}dt\int_{-\infty}^{\infty}\frac{G_{ss}(\omega^2)}{A(\omega^2)}W_1(j\omega)R(-j\omega)e^{j\omega t}d\omega+\\&+C_1+C_2p+\cdots+C_{l-m}p^{l-m-1}+(D_1+D_2p+\cdots+D_{l-m}p^{l-m-1})e^{-pT}.\end{aligned} \tag{17}$$

The solution of Eq. (17) shows that the transfer function of a closed control system depends on p and e^{-pT}. This reduces to the fact that the frequency characteristics corresponding to the system contain undamped frequency components. A graphical method of approximating the amplitude and phase frequency characteristics of the optimum system with the help of the corresponding characteristics of desired, standard, control systems was used in paper [14] to obtain a physically-realizable solution of the problem.

In the present work an analytical method of approximating directly the transfer function $K(p, e^{-pT})$ and not the frequency characteristics is proposed. To obtain the function $K(p)$ in a fraction-rational form, we approximate e^{-pT} by means of Pade's fraction-rational function [16]. In this case we can write that

$$e^x=\lim_{(\mu+\nu)\to\infty}\frac{F_{\mu,\nu}(x)}{G_{\mu,\nu}(x)}, \tag{18}$$

where

$$\begin{aligned}F_{\mu,\nu}(x)&=1+\frac{\nu x}{(\mu+\nu)1!}+\frac{\nu(\nu-1)x^2}{(\mu+\nu)(\mu+\nu-1)2!}+\cdots\\&\cdots+\frac{\nu(\nu-1)\cdots2\times1\times x^{\nu}}{(\mu+\nu)(\mu+\nu-1)\cdots(\nu+1)\mu!}\end{aligned} \tag{19}$$

and

$$\begin{aligned}G_{\mu,\nu}(x)&=1-\frac{\mu x}{(\mu+\nu)1!}+\frac{\mu(\mu-1)x^2}{(\mu+\nu)(\mu+\nu-1)2!}-\cdots\\&\cdots+\frac{(-1)^{\mu}\mu(\mu-1)\cdots2\times1\times x^{\mu}}{(\mu+\nu)(\mu+\nu-1)\cdots(\nu+1)\mu!}.\end{aligned} \tag{20}$$

Taking $x=-pT$ and $\mu=\nu$ so that the moduli of the left-hand and right-hand sides of Eq. (18) be equal for any value of μ, we obtain the following approximate relationships:

for $\mu=\nu=3$

$$e^{-pT}\approx\frac{1-\frac{pT}{2}+\frac{p^2T^2}{10}-\frac{p^3T^3}{120}}{1+\frac{pT}{2}+\frac{p^2T^2}{10}+\frac{p^3T^3}{120}}, \tag{21}$$

for $\mu = \nu = 4$

$$e^{-pT} \approx \frac{1 - \frac{pT}{2} + \frac{3}{28} p^2T^2 - \frac{1}{84} p^3T^3 + \frac{1}{1680} p^4T^4}{1 + \frac{pT}{2} + \frac{3}{28} p^2T^2 + \frac{1}{84} p^3T^3 + \frac{1}{1680} p^4T^4}. \tag{22}$$

The approximate relationships in (21) and (22) are quite adequate for a whole row of practical problems. To evaluate these approximations let us examine the phase frequency characteristics of the left and right sides of Eqs. (21) and (22). The phase frequency characteristic for the function e^{-pT} has the form:

$$\varphi(\omega) = -\omega T, \tag{23}$$

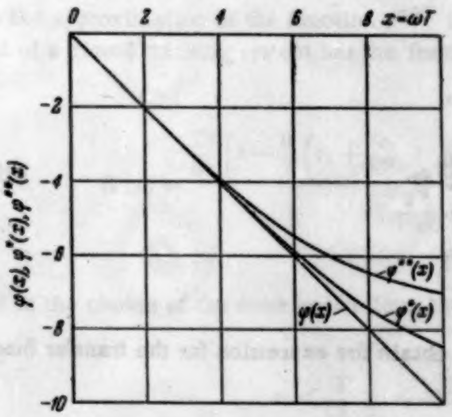
for the right side of (21)

$$\varphi^*(\omega) = \operatorname{arc\,tan} \frac{2\left(\frac{\omega^3T^3}{120} - \frac{\omega T}{2}\right)\left(1 - \frac{\omega^2T^2}{10}\right)}{\left(1 - \frac{\omega^2T^2}{10}\right)^2 - \left(\frac{\omega^3T^3}{120} - \frac{\omega T}{2}\right)^2} \tag{24}$$

and for the right side of (22)

$$\varphi^{**}(\omega) = \operatorname{arc\,tan} \frac{2\left(1 - \frac{3}{28}\omega^2T^2 + \frac{1}{1680}\omega^4T^4\right)\left(\frac{\omega^3T^3}{84} - \frac{\omega T}{2}\right)}{\left(1 - \frac{3}{28}\omega^2T^2 + \frac{1}{1680}\omega^4T^4\right)^2 - \left(\frac{\omega^3T^3}{84} - \frac{\omega T}{2}\right)^2}. \tag{25}$$

The phase frequency characteristics represented by (23), (24), and (25) are shown in the figure. As is apparent from the figure, these characteristics coincide very well in the range of frequencies $0 < \omega T < 6$, which is the most effective in many control systems. Thus, in tracking systems, for example, the indicated range includes, to a first approximation, all the working frequencies up to the cutoff frequency.



The approximate characteristics are different from the optimum ones only to the extent that, for frequencies greater than the cutoff frequency ω_c, they have a smooth shape and do not contain frequency components brought about by the discontinuous nature of the pulse transfer function (13). It is very important to note that the described analytical method of approximation, as opposed to the graphical method in [14], yields good coincidence between the realizable characteristics and the optimum ones, within the working range of frequencies, irrespective of whether the systems being approximated are minimum phase types or not. The final operation in the determination of the optimum characteristic of a linear control system is the substitution of e^{-pT} from Eq. (21) or (22) into Eq. (17), from which we obtain $K(p)$ in the form of a fraction-rational function. In addition, known methods of the synthesis of compensating devices may be utilized with a check of the system's stability.

3. Examples of the Method's Application

1. To determine the optimum transfer function of a tracking system, given $G_{nn}(\omega) = c^2 = 0.612 \cdot 10^{-6}$ sec, $G_{ss}(\omega) = 0$, $W_1(p) = 1$, $T = 1$ sec and given the error coefficients $c_1 = 0.005$ sec and $c_2 = 0.0016$ sec^2, which satisfy the requirements of dynamic accuracy.

In accordance with Eq. (13) the pulse transfer function w (t) has the following form:

$$
\begin{aligned}
w(t) &= A_0 + A_1 t + A_2 t^2 \qquad (0 < t < T),\\
w(t) &= 0 \qquad (0 \geqslant t \geqslant T).
\end{aligned} \tag{26}
$$

Equations (9) will, in this case, be written as

$$
\begin{aligned}
&\int_0^T w(\tau)\, d\tau = 1,\\
&\int_0^T \tau w(\tau)\, d\tau = c_1,\\
&\int_0^T \tau^2 w(\tau)\, d\tau = -c_2.
\end{aligned} \tag{27}
$$

Substitution of (26) into (27) yields the following system of equations:

$$
\begin{aligned}
&A_0 T + A_1 \frac{T^2}{2} + A_2 \frac{T^3}{3} = 1,\\
&A_0 \frac{T^2}{2} + A_1 \frac{T^3}{3} + A_2 \frac{T^4}{4} = c_1,\\
&A_0 \frac{T^3}{3} + A_1 \frac{T^4}{4} + A_2 \frac{T^5}{5} = -c_2,
\end{aligned} \tag{28}
$$

whence

$$
\begin{aligned}
A_0 &= \frac{9}{T} - \frac{36}{T^2} c_1 - \frac{30}{T^3} c_2,\\
A_1 &= -\frac{36}{T^2} + \frac{192}{T^3} c_1 + \frac{180}{T^4} c_2,\\
A_2 &= \frac{30}{T^3} - \frac{180}{T^4} c_1 - \frac{180}{T^5} c_2.
\end{aligned} \tag{29}
$$

Using Eq. (16) and taking into account (26), (29) and (21) we obtain the expression for the transfer function of a closed tracking system:

$$
K(p) = \frac{\frac{T^2}{10}\left[1 - \frac{5}{T}\left(c_1 + \frac{c_2}{T}\right)\right] p^2 + \frac{T}{2}\left(1 - \frac{2}{T} c_1\right) p + 1}{\frac{T^3}{120} p^3 + \frac{T^2}{10} p^2 + \frac{T}{2} p + 1} \tag{30}
$$

The transfer function of the open system is

$$Y(p)=\frac{K(p)}{1-K(p)}=\frac{1}{c_1}\,\frac{\frac{T^2}{10}\left[1-\frac{5}{T}\left(c_1+\frac{c_2}{T}\right)\right]p^2+\frac{T}{2}\left(1-\frac{2}{T}c_1\right)p+1}{p\left[\frac{T^3}{120c_1}p^2+\frac{T}{2}\left(1+\frac{c_2}{Tc_1}\right)\quad p+1\right]}. \qquad (31)$$

For the system to be stable and to have minimum phase angle when in the closed or open condition, it is necessary to impose the following requirements on the error coefficients: $c_1 < \frac{T}{2}$ and $-Tc_1 < c_2 < \frac{T^2}{5} - Tc_1$.

The mean value of the square of the error, determined by the interferences having a spectral density $G_{nn}(\omega) = \epsilon^2$, will have the form

$$\overline{\delta^2_{\beta_n}}=\frac{1}{2\pi}\int_0^\infty |K(j\omega)|^2\epsilon^2 d\omega=\frac{4.5\epsilon^2}{T}\left(1-\frac{8c_1}{T}-\frac{20c_2}{3T^2}+\frac{64c_1^2}{3T^2}+\frac{40c_1c_2}{T^3}+\frac{20c_2^2}{T^4}\right). \qquad (32)$$

Substituting numerical values into (31) we obtain

$$Y(p)=200\,\frac{0.0967p^2+0.495p+1}{p(1.666p^2+0.66p+1)}. \qquad (33)$$

In accordance with (32) we have $\sqrt{\overline{\delta^2_{\beta_n}}}=0.093^\circ$. The systematic tracking error will be determined from the equation

$$\delta_{\beta_c}(t)=c_1\frac{d\beta_i(t)}{dt}+\frac{c^2}{2}\frac{d^2\beta_i(t)}{dt^2}. \qquad (34)$$

Let $\left(\frac{d\beta_i(t)}{dt}\right)_{\max}=\frac{\pi}{5}$ $\sec^{-1}$ and $\left(\frac{d^2\beta_i(t)}{dt^2}\right)_{\max}=\frac{\pi}{12}$ $\sec^{-2}$.

Then, in accordance with (33), the maximum value of the systematic error will be $(\delta\beta_s(t))_{\max} = 0.192^\circ$. Examination of Eq. (33) shows that the tracking system is conditionally stable and is astatic in the first order. It should be noted that if the first three coefficients of the tracking system's error are given, then it is necessary to use the approximation of the function e^{-pT} in accordance with Eq. (22). In this case the optimum transfer function of a closed tracking system has the form:

$$K(p)=\frac{\frac{T^3}{84}\left[1-\frac{9}{T}\left(c_1+\frac{7c_2}{3T}+\frac{14c_3}{9T^2}\right)\right]p^3+\frac{3T^2}{28}\left[1-\frac{14}{3T}\left(c_1+\frac{c_2}{T}\right)\right]p^2+\frac{T}{2}\left(1-\frac{2c_1}{T}\right)p+1}{\frac{T^4}{1680}p^4+\frac{T^3}{84}p^3+\frac{3T^2}{28}p^2+\frac{T}{2}p+1}, \qquad (35)$$

and in the choice of the error coefficients the following conditions must be met:

$$c_1<\frac{T}{2},\quad c_2<\frac{3T^2}{14}-Tc_1,\quad c_3<\frac{T^3}{14}-\frac{9T^2}{14}c_1-\frac{3T}{2}c_2.$$

2. To determine the optimum transfer function of a differentiator, given that $R_{nn}(\tau) = e^{-\alpha|\tau|}$, $G_{nn}(\omega) = \frac{2\alpha}{\omega^2+\alpha^2}$, $\alpha = 20\ \sec^{-1}$, $G_{ss}(\omega) = 0$, $W_i(p) = p$, $T = 10$ sec, $\beta_i(t) = a_0 + a_1 t$ and the systematic differentiation error must be equal to zero. According to Eq. (13) the pulse transfer function is

$$\begin{aligned}&w(t)=A_0+A_1t+C_1\delta(t)+D_1\delta(t-T) && (0<t<T).\\ &w(t)=0 && (0\geqslant t\geqslant T).\end{aligned} \qquad (36)$$

Equations (9), in this case, have the form:

$$\int_0^T w(\tau)\,d\tau = 0, \quad \int_0^T \tau w(\tau)\,d\tau = -1. \tag{37}$$

Substituting (36) into (37) and (12), and treating the last equation as an identity yields the following set of equations:

$$\begin{gathered} A_0 T + A_1 \frac{T^2}{2} + C_1 + D_1 = 0, \quad A_0 \frac{T^2}{2} + A_1 \frac{T^3}{3} + D_1 T = -1, \\ A_0 \alpha - A_1 - \alpha^2 C_1 = 0, \quad A_0 \alpha + A_1 (\alpha T + 1) - \alpha^2 D_1 = 0, \end{gathered} \tag{38}$$

whence

$$\begin{gathered} A_0 = \frac{6\alpha^2}{\alpha^2 T^2 + 6\alpha T + 12}, \quad A_1 = -\frac{12\alpha^2}{T(\alpha^2 T^2 + 6\alpha T + 12)}, \\ C_1 = \frac{6(\alpha T + 2)}{T(\alpha^2 T^2 + 6\alpha T + 12)}, \quad D_1 = -\frac{6(\alpha T + 2)}{T(\alpha^2 T^2 + 6\alpha T + 12)} \end{gathered} \tag{39}$$

Making use of Eqs. (16), (21), (36) and (39) we obtain the optimum transfer function of the differentiator in the form

$$K(p) = p\,\frac{\dfrac{T^2(\alpha T + 12)}{10(\alpha^2 T^2 + 6\alpha T + 12)}\,p^2 - \dfrac{T^3\alpha^2}{10(\alpha^2 T^2 + 6\alpha T + 12)}\,p + 1}{\dfrac{T^3}{120}\,p^3 + \dfrac{T^2}{10}\,p^2 + \dfrac{T}{2}\,p + 1} \tag{40}$$

Equation (40) can be rewritten, after a simple transformation, in the form

$$K(p) = p\,\frac{\dfrac{12(\alpha T + 2)}{T(\alpha^2 T^2 + 6\alpha T + 12)}\,p^2 - \dfrac{12\alpha^2}{\alpha^2 T^2 + 6\alpha T + 12}\,p + \dfrac{120}{T^3}}{\left(p + \dfrac{4.65}{T}\right)\left(p + \dfrac{3{,}675}{T} - j\,\dfrac{3.505}{T}\right)\left(p + \dfrac{3.675}{T} + j\,\dfrac{3.505}{T}\right)}, \tag{41}$$

whence it follows that the control system is stable. After substitution of numerical values of the parameters, Eq. (41) takes the form

$$K(p) = p\,\frac{0.00588p^2 - 0.1165p + 0.12}{(p + 0{,}465)(p + 0.3675 - j \times 0.3505)(p + 0.3675 + j \times 0.3505)}. \tag{42}$$

The response of the differentiator to a unit step function of speed $\beta_1(p) = 1/p^2$ has, according to Eq. (42), the following form:

$$\dot{\beta}_0(t) = 1 - 6{,}64\,e^{-1.465t} + 5.64\,e^{-0.3675t}(\cos 0.3505\,t + 0.0322 \sin 0.3505\,t). \tag{43}$$

If the interference at the input is determined by a function of spectral density as $G_{nn}(\omega) = \epsilon^2$, then the optimum function of the differentiator can be represented in the form

$$K(p) = p\,\frac{1}{\dfrac{T^3}{120}\,p^3 + \dfrac{T^2}{10}\,p^2 + \dfrac{T}{2}\,p + 1}$$

and the mean square differentiation error will be determined by the expression

$$\sqrt{\overline{\sigma^2_{\beta_n}}} = \sqrt{\frac{1}{2\pi}\int_0^\infty |K(j\omega)|^2\varepsilon^2 d\omega} = \sqrt{\frac{6\varepsilon^2}{T^3}}. \tag{44}$$

SUMMARY

1. An approximation of the transcendental function e^{-pT} was used in the work to obtain, with the help of Pade's fraction-rational function, the optimum transfer functions of automatic control systems, determined in [10, 14], directly in a form suitable for engineering application.

2. The cited examples point out the practical applicability of the proposed method of approximation to the solution of problems in the optimization of automatic control systems, in the presence of interference and with given requirements for dynamic accuracy.

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THE INFLUENCE OF LINEAR ZONES AND REGIONS OF SATURATION ON THE DYNAMICS OF TWO-STAGE SERVOMECHANISMS

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The dynamics of the free motion of a two-stage servomechanism are investigated, where the first stage has a relay controlling element whose characteristic contains a loop and a dead region, and the second stage contains a controlling element with a linear zone and with saturation regions. The investigation is carried out by means of the method of point transformations [1-3].

A complete study is made of the nonlinear problem involving the influence on the free oscillations of the servomechanism of the second stage characteristic with the linear zone and the regions of saturation. Analytic expressions are given for the critical values of the basic servomechanism parameters.

I. Posing of the Problem

Two-stage servomechanisms find wide application in automatic regulation and control systems, and in follower systems, for significant amplification of the controlling signal sent from the sensing and transforming devices to the regulating organs. Such servomechanisms are used for amplifying power and torques.

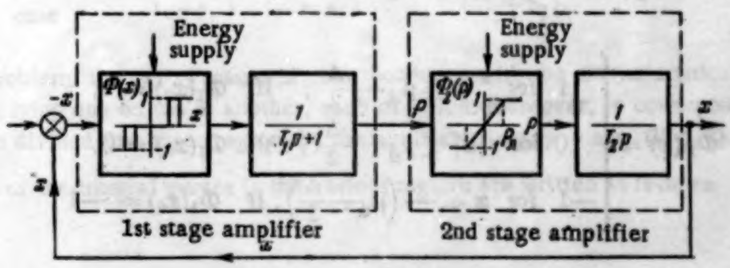

Fig. 1.

The fastest acting servomechanism is obtaned when its controlling element acts with relay, or nearly relay, characteristics. For a given structure, stability may be guaranteed either by introducing dead zones, or by introducing linear portions into the "yes-no" characteristic of the controlling element.

The introduction of dead zones leads to a decrease in the accuracy of the servomechanism; the introduction of linear portions in the relay characteristic allows accuracy to be maintained. Since the size of the linear regions, when the aim is to increase the quality of servomechanism performance, is chosen to be small, when the computations are made for such a servomechanism the compound characteristic of the controlling element is ordinarily replaced by a relay characteristic.

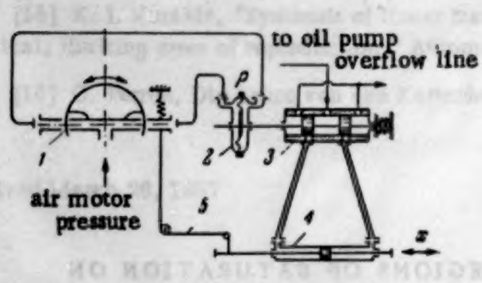

Fig. 2. Schematic of a two-stage pneumo-hydraulic servomechanism: 1) the controlling element (the nozzle-valve type), 2) membrane executive mechanism, 3) the hydraulic valve, 4) the hydraulic servomotor, 5) feedback path.

In the majority of published works, servomechanisms controlled by only one relay are considered, and only certain works [4, 5] are dedicated to two-stage servomechanisms. In the works cited, the second stage approximation of a servomotor is considered as: in [4], a relay characteristic with a dead zone, in [5], a purely linear characteristic.

Here, more typical servomotor characteristics will be considered, characteristics described by uneven nonlinear functions with linear zones and saturation zones. The problem of the present work, from the technological point of view, consists of obtaining recommendataions for the rational construction of two-stage servomechanisms, the block schematic of which is given in Fig. 1. From the mathematical point of view, the problem comes down to the investigation of systems with relay characteristics containing loops, and with nonlinear characteristics with linear and saturation zones. This allows a comparison to be made of the influence of the linear zone ρ_l and the dead zone μ_d on the system's stability and auto-oscillation. The investigation is carried out by the method of point transformations.

As an example of the type of servomechanism considered, one may adduce the pneumo-hydraulic servomechanism (Fig. 2) [4-6].

II. Equations of Motion

The equations describing the free oscillations of the two-stage servomechanism, with the assumptions of [4, 5], but taking into account the saturation zone and the linear portions in the nonlinear characteristic of the controlling element of the second amplification stage, may be written in the following way:

$$T_1\rho + \rho = -\Phi_1(x), \quad T_2\dot{x} = \Phi_2(\rho), \tag{1a}$$

where * ρ is the relative magnitude of the output coordinate of the first amplification stage (relative pressure variation in the housing of the membrane mechanism), $\underline{x}$ is the relative magnitude of the output coordinate of the servomechanism (relative displacement of the servomotor shaft), T_1 is the time for establishing a pulse in the first amplifying stage and T_2 is the response time of the servomotor.

The nonlinear functions, $\Phi_1(x)$ and $\Phi_2(\rho)$, are represented as:

$$\Phi_1(x) = \begin{cases} 1 \text{ for } & x \geqslant \mu_d - \frac{\Delta}{2}, \text{ if } \Phi_1(x_0) = 1, \\ 0 \text{ for } & |x| < \mu_d + \frac{\Delta}{2}, \text{ if } \Phi_1(x_0) = 0, \\ -1 \text{ for } & x \leqslant -\left(\mu_d - \frac{\Delta}{2}\right), \text{ if } \Phi_1(x_0) = -1 \end{cases} \tag{1b}$$

(where x_0 is the initial condition at time t = +0),

$$\Phi_2(\rho) = \begin{cases} 1 \text{ for } & \rho \geqslant \rho_l, \\ \rho \text{ for } & -\rho_l < \rho < \rho_l, \\ -1 \text{ for } & \rho \leqslant -\rho_l, \end{cases} \tag{1c}$$

* Since we are considering the free oscillations of the servomechanism, the input signal, the rotation of the manifold with nozzle with respect to the valve, equals zero, and the opening of the pneumo-relay valve depends only on the action of the feedback path (5).

where Δ, μ_d and ρ_l are relative quantities for the slack in, respectively, the feedback path, the dead zone and the linear region of the relay.

III. Salient Features of (ρ, x) Phase Space

The (ρ, x) phase space (plane) in the problem under consideration differs from the (ρ, x) phase plane considered in [4, 5].

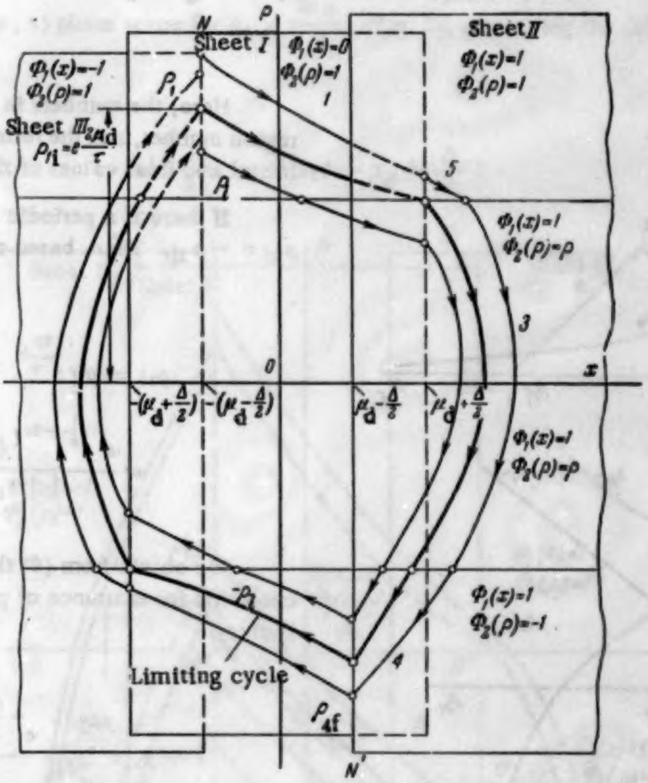

Fig. 3. Three-sheeted (ρ, x) phase plane and limiting cycle for the case $1-\rho_l e^{\frac{2\mu_d}{\tau}} = e^{-\frac{\Delta}{\tau}} - 2\rho_l(1+\rho_l)$

In the given problem, the (ρ, x) space, in correspondence with the characteristics of $\Phi_1(x)$, is a plane composed of three sheets lying one on top of another, each of which, moreover, in correspondence with the characteristics of $\Phi_2(\rho)$, is divided into three regions by the auxiliary lines, $\rho = \pm\rho_l$ (Fig. 3).

The equations of the integral curves in the various regions are written as follows:

in region 1

$$\tau \ln \frac{\rho_{1f}}{\rho_{1i}} = -2\mu_d; \tag{2}$$

where $\tau = \frac{T_1}{T_2}$;

in region 5

$$\tau[\ln(1+\rho_l) - \ln(1+\rho_{1f})] = \mu_d + \frac{\Delta}{2} - x_{5f}; \tag{3}$$

in region 3

$$\tau\{[-\rho_l - \ln(1-\rho_l)] - [\rho_l - \ln(1+\rho_l)]\} = -x_{4f} + x_{3f}; \tag{4}$$

in region 4

$$\tau[\ln(1+\rho_{4f}) - \ln(1-\rho_l)] = \mu_d - \frac{\Delta}{2} - x_{3f}. \tag{5}$$

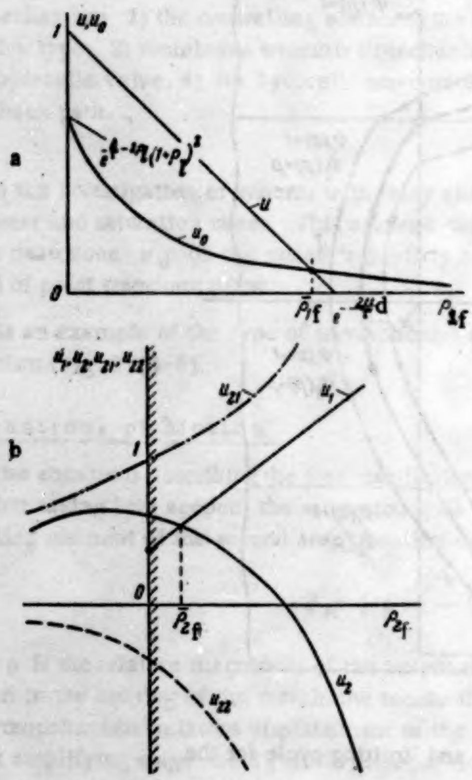

Fig. 4. Diagrams of point transformations.

Here, the numbers in the subscripts refer to the region number, and the letters "i" and "f" refer to the initial and final values of the coordinates in the region.

If there is a periodic oscillation in the system, $\rho_{4f} = -\rho_{1i}$. Then, based on Eqs. (2)-(5), we have

$$\rho_{1i} = \rho_{1f} e^{\frac{2\mu_d}{\tau}}, \quad \rho_{4f} = -\rho_{1i} = \frac{e^{-\frac{\Delta}{\tau} - 2\rho_l}(1+\rho_l)^2}{1+\rho_{1f}} - 1. \tag{6}$$

We obtain from (6) the equation expressing the condition for existence of periodic oscillations in the system:

$$1 - \rho_{1f} e^{\frac{2\mu_d}{\tau}} = \frac{e^{-\frac{\Delta}{\tau} - 2\rho_l}(1+\rho_l)^2}{1+\rho_{1f}} \tag{7}$$

If Eq. (7) has real and positive roots, ρ_{1f}, then there exist periodic oscillations in the system. We introduce the notation

$$1 - \rho_{1f} e^{\frac{2\mu_d}{\tau}} = u(\rho_{1f}),$$
$$\frac{e^{-\frac{\Delta}{\tau} - 2\rho_l}(1+\rho_l)^2}{1+\rho_{1f}} = u_0(\rho_{1f}). \tag{8}$$

The curves of the functions $u(\rho_{1f})$ and $u_0(\rho_{1f})$, represented in Fig. 4a, have only one point of intersection for

$$\rho_{1f} = \bar{\rho}_{1f} = \frac{e^{-\frac{2\mu_d}{\tau}} - 1}{2} + \left\{\frac{(e^{-\frac{2\mu_d}{\tau}} - 1)^2}{4} + e^{-\frac{2\mu_d}{\tau}}\left[1 - e^{-\frac{\Delta}{\tau} - 2\rho_l}(1+\rho_l)^2\right]\right\}^{1/2} \tag{9}$$

If the relationship of the system parameters is such that

$$1-\rho_l e^{\frac{2\mu_d}{\tau}} = e^{-\frac{\Delta}{\tau}-2\rho_l}(1+\rho_l), \tag{10}$$

then $\rho_{1l} = \bar{\rho}_{1f}$, and a limiting cycle is set up on the phase plane, represented in Fig. 3, and auto-oscillation arises in the system.

1. We consider the (ρ, x) phase space for $\rho_l < \frac{2\mu_d}{\tau}$ (Fig. 5), containing the following integral curves:

in region 1

$$\tau \ln \frac{\rho_l}{\rho_{l1}} = -x_{1f} - \mu_d + \frac{\Delta}{2}; \tag{11}$$

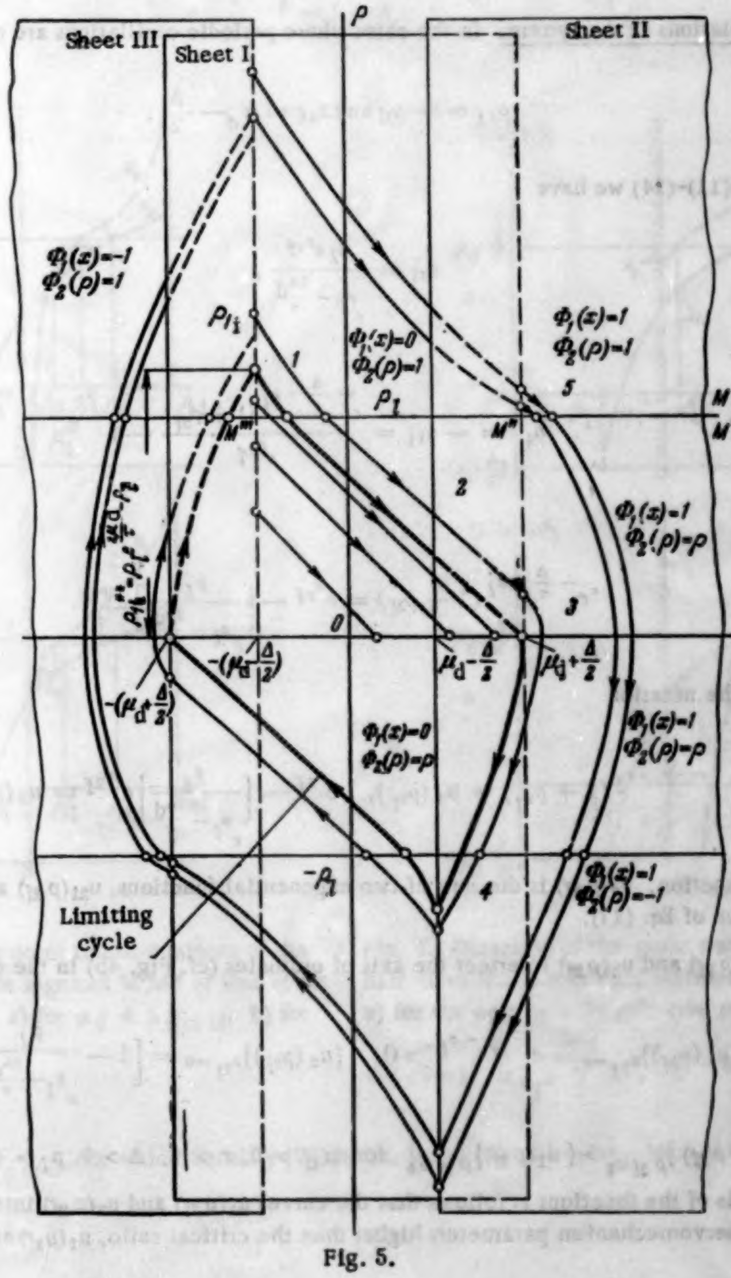

Fig. 5.

in region 2

$$\tau(\rho_{2f} - \rho_l) = -\mu_d - \frac{\Delta}{2} + x_{1f}; \tag{12}$$

in region 3

$$\tau\{[-\rho_l - \ln(1-\rho_l)] - [\rho_{2f} - \ln(1+\rho_{2f})]\} = -x_{3f} + \mu_d + \frac{\Delta}{2}; \tag{13}$$

in region 4

$$\tau[\ln(1+\rho_{4f}) - \ln(1-\rho_l)] = \mu_d - \frac{\Delta}{2} - x_{3f}. \tag{14}$$

<u>Periodic oscillations in the system.</u> In the cases where periodic oscillations are present

$$\rho_{4f} = -\rho_{1i} \text{ and } x_{4f} = \mu_d - \frac{\Delta}{2}. \tag{15}$$

Based on Eqs. (11)-(14) we have

$$\rho_{1i} = \frac{\rho_l e^{\rho_{2f}}}{e^{\rho_l - \frac{2\mu_d}{\tau}}}, \tag{16a}$$

$$\rho_{4f} = -\rho_{1i} = \frac{e^{-\frac{\Delta}{\tau} - \rho_l}(1+\rho_{2f})}{e^{\rho_{2f}}} - 1 \tag{16b}$$

or

$$e^{-\frac{\Delta}{\tau} - \rho_l}(1+\rho_{2f}) = e^{\rho_{2f}} - \left[\frac{\rho_l}{e^{\rho_l - \frac{2\mu_d}{\tau}}}\right] e^{2\rho_{2f}}. \tag{17}$$

We introduce the notation

$$e^{-\frac{\Delta}{\tau} - \rho_l}(1+\rho_{2f}) = u_1(\rho_{2f}), \quad e^{\rho_{2f}} - \left[\frac{\rho_l}{e^{\rho_l - \frac{2\mu_d}{\tau}}}\right] e^{2\rho_{2f}} = u_2(\rho_{2f}) \tag{18}$$

[$u_1(\rho_{2f})$ is a linear function, $u_2(\rho_{2f})$ is the sum of two exponential functions, $u_{21}(\rho_{2f})$ and $u_{22}(\rho_{2f})$], and we shall seek a graphic solution of Eq. (17).

The curves $u_1(\rho_{2f})$ and $u_2(\rho_{2f})$ intersect the axis of ordinates (cf. Fig. 4b) in the points

$$\{u_1(\rho_{2f})\}_{\rho_{2f}=0} = e^{-\frac{\Delta}{\tau} - \rho_l} > 0, \quad \{u_2(\rho_{2f})\}_{\rho_{2f}=0} = \left[1 - \frac{\rho_l}{e^{\rho_l - \frac{2\mu_d}{\tau}}}\right] > 0.$$

With this, $\{u_2(\rho_{2f})\}_{\rho_{2f}=0} > \{u_1(\rho_{2f})\}_{\rho_{2f}=0}$ for $\mu_d > 0, \tau > 0, \Delta > 0, \rho_l > 0$.

From an analysis of the functions it follows that the curves $u_1(\rho_{2f})$ and $u_2(\rho_{2f})$ intersect in only one point. For each ratio of the servomechanism parameters higher than the critical ratio, $u_1(\rho_{2f})$ and $u_2(\rho_{2f})$ intersect only once

$(\rho_{2f} = \bar{\rho}_{2f})$, i. e., for the case $\rho_l < \frac{2\mu_d}{\tau}$, periodic oscillations can arise in the system only with a definite amplitude $\bar{\rho}_{1l}$.

Critical ratio of the parameters. It follows from system Eq. (16a) that $\rho_{2f} = 0$ for $\rho_{1l} = \rho_{1l}^{**} = \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}$, where $\rho_{1l}^{**} > \rho_l$ since $\rho_l < \frac{2\mu_d}{\tau}$.

This indicates that the boundary of the dead zone in the first relay is reached at the moment of switching in of the second controlling element.

Thus, a necessary condition for the existence of periodic oscillations is the holding of the inequality

$$\rho_{1l} \geqslant \rho_{1l}^{**} \ (\text{or} \ \ \rho_{2f} \geqslant 0). \tag{19}$$

Substituting $\rho_{2f} = 0$ in Eq. (17), we obtain the following critical ratio of the parameters:

$$e^{-\frac{\Delta}{\tau} - \rho_l} = 1 - \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}. \tag{20}$$

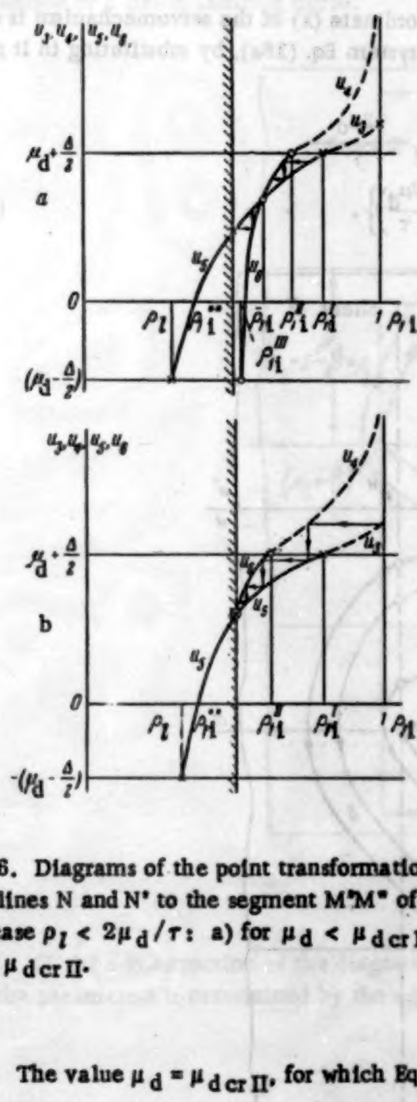

Fig. 6. Diagrams of the point transformations of the half lines N and N' to the segment M"M"' of line M for the case $\rho_l < 2\mu_d/\tau$: a) for $\mu_d < \mu_{d\,cr\,II}$, b) for $\mu_d = \mu_{d\,cr\,II}$.

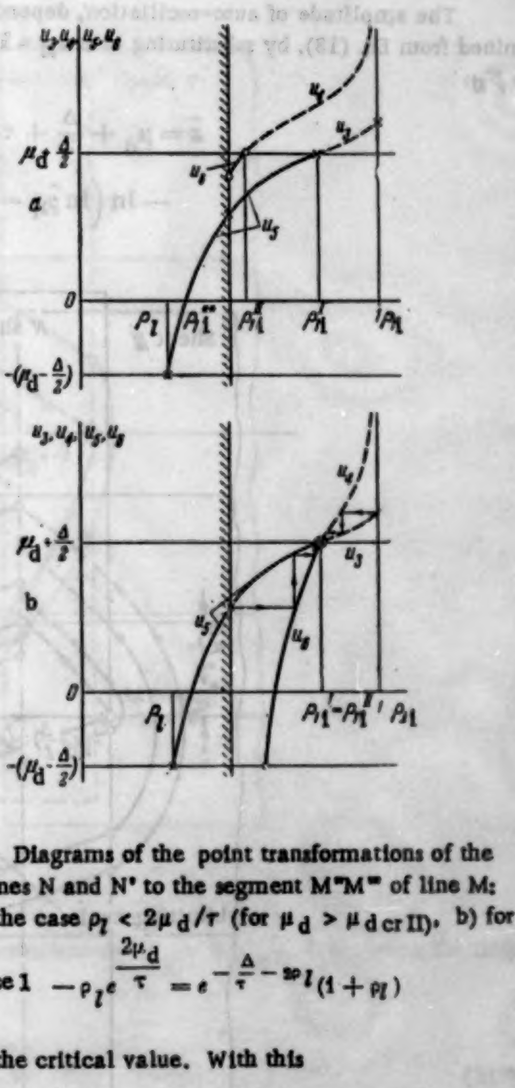

Fig. 7. Diagrams of the point transformations of the half-lines N and N' to the segment M"M"' of line M: a) for the case $\rho_l < 2\mu_d/\tau$ (for $\mu_d > \mu_{d\,cr\,II}$), b) for the case $1 - \rho_l e^{\frac{2\mu_d}{\tau}} = e^{-\frac{\Delta}{\tau} - 2\rho_l}(1 + \rho_l)$

The value $\mu_d = \mu_{d\,cr\,II}$, for which Equality (20) holds, is the critical value. With this

$$e^{\frac{2\mu_{d\,cr\,II}}{\tau}} = \frac{1 - e^{-\frac{\Delta}{\tau} - \rho_l}}{\rho_l^* e^{-\rho_l}}.$$

The ratio of the parameters determining the boundary of the qualitatively different cases of servomechanism motion has the form:

$$e^{-\frac{\Delta}{\tau} - \rho_l} \leqslant 1 - \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}. \tag{21}$$

A. The construction of the diagrams of the point transformation for $\mu_d < \mu_{d\,cr\,II}$ (for the case $\rho_l < \frac{2\mu_d}{\tau}$) is considered in Appendix I.

The curves of the functions $u_3(\rho_{1l})$, $u_4(\rho_{1l})$, $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$ (Fig. 6a) have one point of intersection $\bar{\rho}_{1l}$, and the resulting limiting cycle will be stable.

The amplitude of auto-oscillation, depending on the output coordinate ($\bar{x}$) of the servomechanism is determined from Eq. (13), by substituting in it $x_{3f} = \bar{x}$, $\rho_{2f} = 0$, and from system Eq. (16a), by substituting in it $\rho_{1l} = \bar{\rho}_{1l}$:

$$\bar{x} = \mu_d + \frac{\Delta}{2} + \tau\Big\{\ln\bar{\rho}_{1l} - \ln\rho_l + \rho_l - \frac{2\mu_d}{\tau} - \\ - \ln\Big(\ln\bar{\rho}_{1l} - \ln\rho_l + 1 + \rho_l - \frac{2\mu_d}{\tau}\Big)\Big\}. \tag{22}$$

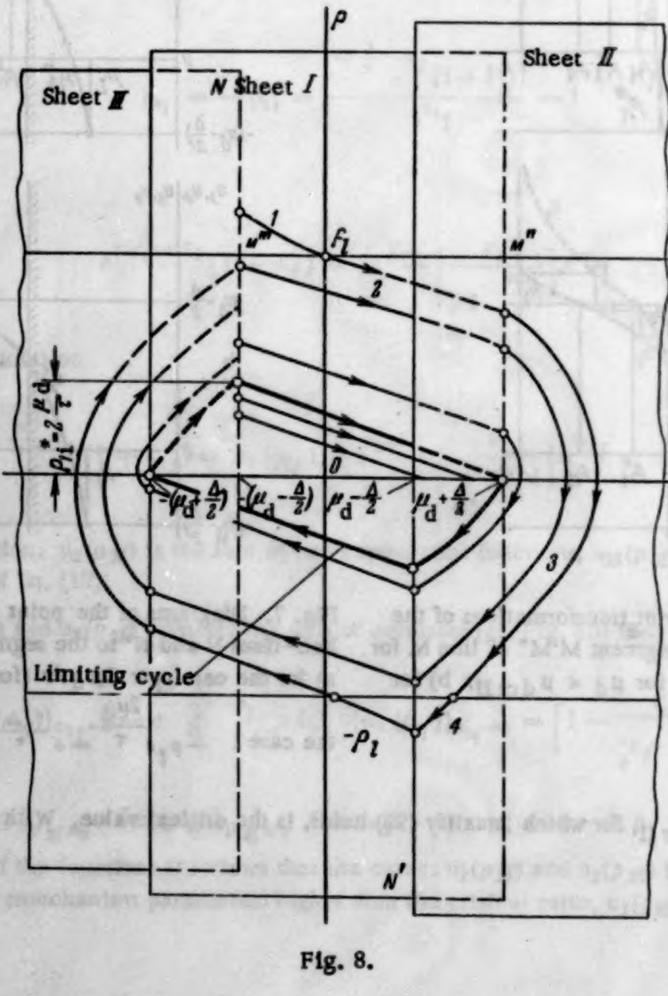

Fig. 8.

B. **Construction of the diagrams of the point transformation for $\mu_d = \mu_{d\,cr\,II}$, i. e., when the ratio of the parameters is determined from the equation**

$$e^{-\frac{\Delta}{\tau}-\rho_l} = 1-\rho_l e^{\frac{2\mu_d}{\tau}-\rho_l} \qquad (21')$$

is considered in Appendix II.

It follows from the diagrams of the point transformations (Fig. 6b) that curves $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$ intersect for $\rho_{1l} = \rho^{**}_{1l}$, i. e., for the limiting value of ρ_{1l} for which periodic oscillations of the system are possible (for $\rho_l < \frac{2\mu_d}{\tau}$).

In this case, the limiting cycle formed in phase space (Fig. 5), as follows from the diagram of the point transformation (Fig. 6b), is semistable.

This latter means that the least variation of the parameter ratio to one side or another (of the critical ratio) will lead either to the destruction of the periodic motion and the damping of oscillation or, conversely, to the formation of a stable limiting cycle.

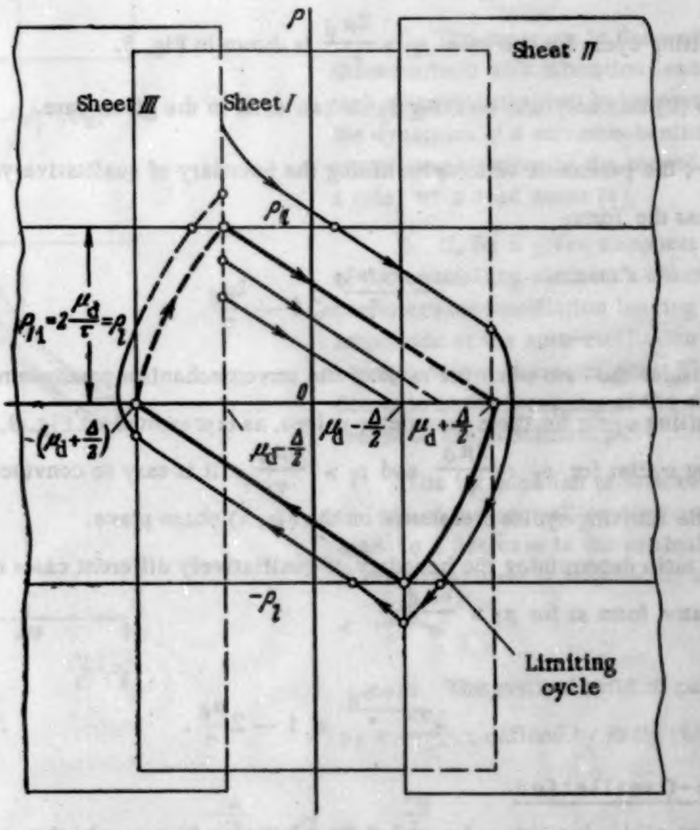

Fig. 9.

C. **The construction of the diagrams of the point transformations for $\mu_d > \mu_{d\,cr\,II}$, i. e., when the ratio of the parameters is determined by the equation**

$$e^{-\frac{\Delta}{\tau}-\rho_l} > 1-\rho_l e^{\frac{2\mu_d}{\tau}-\rho_l} \qquad (21'')$$

is considered in Appendix III.

Figure 7a shows the relative placement of the curves $u_3(\rho_{1I})$, $u_4(\rho_{1I})$, $u_5(\rho_{1I})$ and $u_6(\rho_{1I})$, allowing the conclusion to be drawn that, for $\mu_d > \mu_{d\,cr\,II}$, periodic oscillations do not arise in the system.

For $\mu_d > \mu_{d\,cr\,II}$ the system, after an arbitrary initial deviation, moves to an equilibrium position.

D. The construction of the diagrams of the point transformations in the case when the ratio of the system parameters is determined by Eq. (10) is investigated in Appendix IV.

Figure 7b shows the relative placement of the curves $u_3(\rho_{1I})$, $u_4(\rho_{1I})$, $u_5(\rho_{1I})$ and $u_6(\rho_{1I})$, allowing the conclusion to be drawn that, if the system parameters are connected by Relationship (10), then a stable limiting cycle is formed on the phase plane, and auto-oscillation is established in the system.

2. We consider the (ρ, x) phase space for $\rho_l > \frac{2\mu_d}{\tau}$. It would seem (from a comparison with the case $\rho_l < \frac{2\mu_d}{\tau}$) that, with this ratio of the servomechanism parameters, the least value of the initial deviation of the parameter ρ for which auto-oscillation is possible, must satisfy the inequality $\rho^*_{1I} < \rho_l$. However, as investigation showed [5]

$$\rho^*_{1I} = \frac{2\mu_d}{\tau}. \tag{23}$$

The semi-stable limiting cycle for the case $\rho_l > \frac{2\mu_d}{\tau}$ is shown in Fig. 8.

It follows from [5] that only one limiting cycle can exist in the given case.

For $\rho_l > \frac{2\mu_d}{\tau}$, the parameter ratio determining the boundary of qualitatively different cases of servomechanism motion has the form:

$$e^{-\frac{\Delta+2\mu_d}{\tau}} \leqslant 1 - \frac{2\mu_d}{\tau}. \tag{24}$$

3. We now consider the case when the ratio of the servomechanism parameters is determined by the equality $\rho_l = \frac{2\mu_d}{\tau}$. The limiting cycle for these parameter values, as represented on Fig. 9, is a "bounding" one with respect to the limiting cycles for $\rho_l < \frac{2\mu_d}{\tau}$ and $\rho_l > \frac{2\mu_d}{\tau}$. It is easy to convince oneself of this by considering the poisition of the limiting cycles mentioned on the (ρ, x) phase plane.

The parameter ratio determining the boundary of qualitatively different cases of servomechanism motionfor $\rho_l = \frac{2\mu_d}{\tau}$ has the same form as for $\rho_l > \frac{2\mu_d}{\tau}$:

$$e^{-\frac{\Delta+2\mu_d}{\tau}} \leqslant 1 - 2\frac{\mu_d}{\tau}. \tag{25}$$

IV. System Auto-Oscillation

The investigation of the dynamics of a two-stage relay-type servomechanism containing relay functions with loops and uneven nonlinear functions with linear and saturation zones showed that it is possible to have stable auto-oscillation in the system if $\rho_l > \frac{2\mu_d}{\tau}$ for $\mu_d < \mu_{d\,cr\,I}$ $\left[\mu_{d\,cr\,I}\right.$ is defined by the equation $\exp\left(-\frac{\Delta + 2\mu_{d\,cr\,I}}{\tau}\right) = 1 - \left.\frac{2\mu_{d\,cr\,I}}{\tau}\right]$ or if $\rho_l < \frac{2\mu_d}{\tau}$ for $\mu_d < \mu_{d\,cr\,II}$ $\left[\mu_{d\,cr\,II}\right.$ is defined by the equation $\exp\left(-\frac{\Delta}{\tau} - \rho_l\right) = 1 - \rho_l \exp\left(\frac{2\mu_{d\,cr\,II}}{\tau} - \rho_l\right) \Big]$

If in the servomechanism the magnitude of the linear zone is $\rho_l > \frac{2\mu_d}{\tau}$, then for $\mu_d < \mu_{d\,cr\,I}$, a limiting cycle is formed on the phase plane inside the linear zone. The amplitude of auto-oscillation is $\bar{\rho}_{1I} < \rho_l$. If however $\mu_d \ll \mu_{d\,cr\,I}$, then the amplitude of auto-oscillation is $\bar{\rho}_{1I} > \rho_l$.

If the width of the servomechanism's linear zone is $\rho_l < \frac{2\mu_d}{\tau}$ then, for $\mu_d < \mu_{d\,cr\,I}$, a limiting cycle is not formed inside the linear zone on the phase plane, since the previous trajectory, with initial deviation $\rho_{1l} \le \rho_l$, will tend to a quiescent segment, and auto-oscillation may arise only outside the linear zone. If now $\mu_d > \mu_{d\,cr\,II}$, there will be no limiting cycle even external to the linear zone (for $\rho_{1l} > \rho_l$) and all trajectories will be deflected to the quiescent segment for any initial deviation, ρ_{1l}.

And, finally, if $\mu_d < \mu_{d\,cr\,II}$, a limiting cycle will be formed leaving the linear zone.

SUMMARY

1. A two-stage servomechanism with a relay characteristic with loops in the first amplifying stage an uneven nonlinear characteristic with linear and saturation zones in the second amplifying stage has two critical parameter ratios.

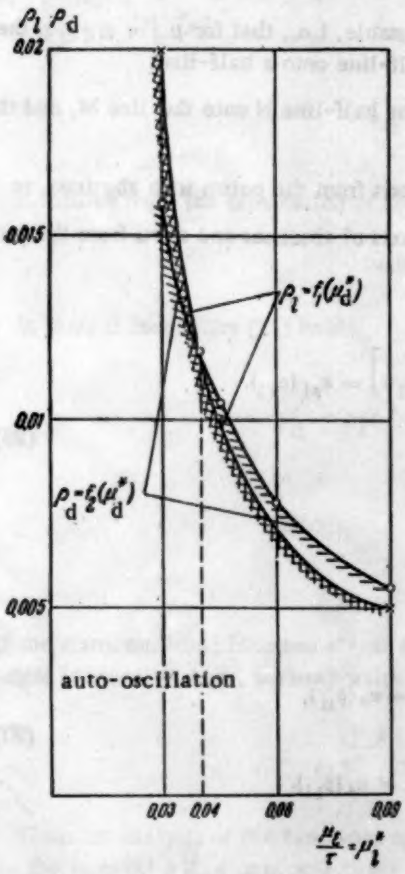

Fig. 10.

In the case when the width of the linear zone is $\rho_l > \frac{2\mu_d}{\tau}$, the critical parameter ratio has the same form as in [5]. If the width of the linear zone is $\rho_l < \frac{2\mu_d}{\tau}$, then the critical ratio has the form of (20).

2. The presence in the second stage of a nonlinear characteristic with saturation leads to the dynamics of such a servomechanism being essentially different from the dynamics of a servomechanism with a purely linear controlling element in the second stage [5], or one with a relay with dead zones [4].

3. If, for a given steepness of the linear portion of the controlling element's characteristic, the system possesses auto-oscillation leaving this zone, then the amplitude of the auto-oscillation will be less than for a purely linear characteristic. This is explained by the decrease in the steepness of the characteristic with increase of the coordinate ρ.

The introduction of saturation regions in the characteristic of the controlling element of the second stage leads to a decrease in the critical value of the dead zone, if the linear zone is selected by the condition that $\rho_l < \frac{2\mu_d}{\tau}$.

4. The critical ratio of parameters in the case $\rho_l < \frac{2\mu_d}{\tau}$, defined by Ratio (20), has the form:

$$e^{-\frac{\Delta}{\tau}} = e^{\rho_l} - \rho_l e^{\frac{2\mu_d}{\tau}}.$$

There was obtained in [4] the critical parameter ratio (Eq. (3.13)) for a similar structure of a servomechanism, this one, however, having a relay controlling element in the second stage with a dead zone ρ_d. This ratio is defined by the equality $\rho_d = \dfrac{e^{\frac{\Delta}{\tau}} - 1}{e^{\frac{2\mu_d+\Delta}{\tau}} - 1}$.

It follows from the dependence of the critical values of ρ_l and ρ_d on $\mu_d^* = \frac{\mu_d}{\tau}$, for $\Delta^* = \frac{\Delta}{\tau} = 0.001$,

as given in Fig. 10, that the linear zone narrows the region of auto-oscillation for values of the first-stage dead zone of $\mu_d > 0.04$.

For $\mu_d < 0.04$, the second-stage linear zone broadens the region of auto-oscillation, in comparison with the auto-oscillation region for the dead zone ρ_d.

The present work was carried out under the direction of V. V. Petrov.

Appendix I

We consider the diagrams of the point transformation for $\mu_d < \mu_{d\,cr\,II}$ (for the case $\rho_l < \frac{2\mu_d}{\tau}$). We show that here the equilibrium position is stable when the initial deviation does not exceed a definite value (for example, $\rho_{1i} > \rho_{1i}^{**} = \rho_l e^{2\mu_d/\tau - \rho_l}$ for $x = -\left(\mu_d - \frac{\Delta}{2}\right)$).

We shall also show that when periodic motions occur they will be stable, i.e., that for $\mu_d < \mu_{d\,cr\,II}$ the initial deviation exceeds ρ_{1i}. For this, we consider the mapping of a half-line onto a half-line.

For the given case, it is expedient to study the transformation of the half-line N onto the line M, and the half-line N' onto the same half-line (Fig. 5).

The half-lines N and N' are parallel to the axis of ordinates and start from the points with abscissas, respectively, $-\left(\mu_d - \frac{\Delta}{2}\right)$ and $\mu_d + \frac{\Delta}{2}$. The line M is parallel to the axis of abscissas and starts from the point with ordinate ρ_l. On the basis of Eqs. (3)-(6) we have

$$x_{3i} = \mu_d + \frac{\Delta}{2} + \tau\left[\ln\left(1 + \rho_{1i} e^{-\frac{2\mu_d}{\tau}}\right) - \ln(1 + \rho_l)\right] = x_{3i}(\rho_{1i}),$$
$$x_{3l} = \mu_d - \frac{\Delta}{2} - \tau \ln \frac{1 + \rho_{4f}}{1 + \rho_l} - 2\tau\rho_l = x_{3l}(\rho_{4f}). \tag{26}$$

Imposing Condition (15) on Eqs. (26) we obtain

$$\mu_d + \frac{\Delta}{2} + \tau \ln \frac{1 + \rho_{1i} e^{-\frac{2\mu_d}{\tau}}}{1 + \rho_l} = x_{3i}(\rho_{1i}) = u_3(\rho_{1i}),$$
$$\mu_d - \frac{\Delta}{2} - \tau \ln \frac{1 - \rho_{1i}}{1 + \rho_{1i}} - 2\tau\rho_l = \bar{x}_{3i}(\rho_{1i}) = u_4(\rho_{1i}). \tag{27}$$

If Condition (21) is met, the functions $u_3(\rho_{1i})$ and $u_4(\rho_{1i})$ are the expressions for the functions corresponding to the point transformations of half-line N and half-line N' to the segment M'M" of line M.

We now consider the mutual placement of the curves of $u_3(\rho_{1i})$ and $u_4(\rho_{1i})$ (Fig. 6a).

It was previously proved that, for $\rho_l < \frac{2\mu_d}{\tau}$, periodic oscillations could exist in the system only in case $\rho_{1i} \geqslant \rho_{1i}^{**} = \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}$ and the parameter ratio satisfied Condition (21).

For $\rho_{1i} = 1$, we get

$$u_3(\rho_{1l}) = \mu_d + \frac{\Delta}{2} + \tau \ln \frac{1 + e^{-\frac{2\mu_d}{\tau}}}{1 + \rho_l}, \quad u_4(\rho_{1l}) = \infty,$$

from which we conclude that if Inequality (21) holds and for $\rho_l < \dfrac{2\mu_d}{\tau}$,

$$\mu_d + \frac{\Delta}{2} < \{u_3(\rho_{1l})\}_{\rho_{1l}-1} < \{u_4(\rho_{1l})\}_{\rho_{1l}-1}.$$

When the initial deviation of coordinate ρ equals $\rho_{1l}^{I} = \rho_l e^{\frac{2\mu_d}{\tau}}$, the function $u_3(\rho_{1l})$ attains its limiting value, $\{u_3(\rho_{1l})\}_{\rho_{1l}-\rho_l e^{\frac{2\mu_d}{\tau}}} = \mu_d + \dfrac{\Delta}{2}$. The function $u_4(\rho_{1l})$ attains its limiting value, equal to $\mu_d + \dfrac{\Delta}{2}$, for

$$\rho_{1l} = \rho_{1l}^{II} = 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l}(1 + \rho_l).$$

It follows from the orientation of the curves of $u_3(\rho_{1l})$ and $u_4(\rho_{1l})$ that

$$\{u_4(\rho_{1l})\}_{\rho_{1l}-\rho_{1l}^{I}} > \{u_3(\rho_{1l})\}_{\rho_{1l}-\rho_{1l}^{I}}.$$

In fact, if Inequality (21) holds,

$$\rho_{1l}^{I} = \rho_l e^{\frac{2\mu_d}{\tau}} > 1 - e^{-\frac{\Delta}{\tau} - \rho_l} > \rho_l e^{\frac{2\mu_d}{\tau} - \mu_d} = \rho_{1l}^{**}$$

or

$$\rho_l e^{\frac{2\mu_d}{\tau}} > 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l} e^{\rho_l},$$

and if the transcendental function e^{ρ_l} is expanded in a series, limited just to the first two terms, we obtain only a stronger inequality, i. e., we may write

$$\rho_l e^{\frac{2\mu_d}{\tau}} > 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l}(1 + \rho_l) = \rho_{1l}^{II}.$$

Thus, an analysis of the functions $u_3(\rho_{1l})$ and $u_4(\rho_{1l})$ shows that the curves $u_3(\rho_{1l})$ and $u_4(\rho_{1l})$ do not intersect in the interval $\rho_{1l}^{II} < \rho_{1l} < 1$ if the parameter ratio satisfies Condition (21).

The point transformations of the half-lines N and N' to the segment M"M" of line M will be carried out by means of functions $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$ which, based on (11)-(15), may be written in the form

$$-\mu_d + \frac{\Delta}{2} - \tau \ln \frac{\rho_l}{\rho_{1l}} = x_{lf}(\rho_{1l}) = u_5(\rho_{1l}),$$

$$\left.\begin{aligned} &\mu_d + \frac{\Delta}{2} + \tau(\rho_{2f} - \rho_{1l}) = x_{lf}(\rho_{1l}, \rho_{2f}) \\ &\frac{1 - \rho_{1l}}{1 + \rho_{2f}} = e^{-\frac{\Delta}{\tau} - \rho_l - \rho_{2f}} \end{aligned}\right\} = u_6(\rho_{1l}). \tag{28}$$

We now determine the placement of curves $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$, as described by Eqs. (28).

Since it was previously proved that only one limiting cycle can exist in the system, curves $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$ can intersect in only one point.

Moreover, it was proved that periodic oscillation of the system (for $\rho_l < \frac{2\mu_d}{\tau}$) can exist only in case $\rho_{1l} \geqslant \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}$ and the parameter ratio satisfies Inequality (21).

Functions $u_3(\rho_{1l})$ and $u_5(\rho_{1l})$ are identically equal for $\rho_{1l} = \rho_{1l}^{I}$.

Functions $u_4(\rho_{1l})$ and $u_6(\rho_{1l})$ are identically equal for $\rho_{1l} = \rho_{1l}^{II}$.

With an initial deviation of $\rho_{1l} = \rho_{1l}^{**} = \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}$ the function $u_5(\rho_{1l})$ equals $\mu_d + \frac{\Delta}{2} - \tau\rho_l$.

At $\rho_{1l} = \rho_l$ the function $u_5(\rho_{1l})$ attains its limiting value, $\{u_5(\rho_{1l})\}_{\rho_{1l}=\rho_l} = -\left(\mu_d - \frac{\Delta}{2}\right)$.

The function $u_6(\rho_{1l})$ attains the same value, $-\left(\mu_d - \frac{\Delta}{2}\right)$ for

$$\rho_{1l} = \rho_{1l}^{III} = 1 - e^{-\frac{\Delta}{\tau} - \rho_l} e^{-\rho_l + \frac{2\mu_d}{\tau}}\left(1 + \rho_{1l} - \frac{2\mu_d}{\tau}\right). \tag{29}$$

It follows from Eq. (29) that

$$\rho_{1l}^{III} > 1 - e^{-\frac{\Delta}{\tau} - \rho_l},$$

since, if we take the expression $(1 + \rho_{1l} - \frac{2\mu_d}{\tau})$ as an approximation to the transcendental function $e^{\rho_{1l} - \frac{2\mu_d}{\tau}}$ by means of a series carried to only two terms, then

$$\rho_{1l}^{III} \approx 1 - e^{-\frac{\Delta}{\tau} - \rho_l}$$

Then if Inequality (21) holds,

$$\rho_{1l}^{III} > \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l} \text{ and } \rho_{1l}^{III} > \rho_{1l}^{**} > \rho_l.$$

Thus, the orientation of the curves $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$ allows us to conclude that the curves have a point of intersection for $\rho_{1l} = \bar{\rho}_{1l}$, and that the resulting limiting cycle will be stable.

Appendix II

We now consider the diagram of the point transformations for $\mu_d = \mu_{d\,cr\,II}$ (Fig. 6b), i. e., when the parameter ratio is defined by Eq. (21').

The mapping of the half-lines N and N' into the segment M'M" of line M is carried out by means of the functions $u_3(\rho_{1l})$ and $u_4(\rho_{1l})$, described by the system of Eqs. (27).

For $\rho_{1l} = 1$, in the case when the Equality (21') holds, and for $\rho_l < \frac{2\mu_d}{\tau}$, we have

$$\mu_d + \frac{\Delta}{2} < \{u_3(\rho_{1l})\}_{\rho_{1l}=1} < \{u_4(\rho_{1l})\}_{\rho_{1l}=1} = \infty.$$

With an initial deviation of ρ_{1l}, equal to $\rho_{1l}^{I} = \rho_l e^{\frac{2\mu_d}{\tau}}$, the function $u_3(\rho_{1l})$ attains its limiting value, $(\mu_d + \frac{\Delta}{2})$. The function $u_4(\rho_{1l})$ attains its limiting value, equal to $(\mu_d + \frac{\Delta}{2})$, for ρ_{1l}, equal to $\rho_{1l}^{II} = 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l}(1 + \rho_l)$.

From the placement of curves $u_3(\rho_{1l})$ and $u_4(\rho_{1l})$ it follows that

$$\{u_4(\rho_{1l})\}_{\rho_{1l}=\rho^I_{1l}} > \{u_3(\rho_{1l})\}_{\rho_{1l}=\rho^I_{1l}},$$

since, if Equality (21') holds,

$$\rho^I_{1l} = \rho_l e^{\frac{2\mu_d}{\tau}} > 1 - e^{-\frac{\Delta}{\tau} - \rho_l} = \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l} = \rho^{**}_{1l}$$

or

$$\rho^I_{1l} > 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l} e^{\rho_l} = \rho^{**}_{1l},$$

and if the function e^{ρ_l} is expanded in a series limited to only two terms, we obtain only a stronger inequality, i.e.,

$$\rho^I_{1l} > 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l} (1 + \rho_l) = \rho^{II}_{1l} < \rho^{**}_{1l}.$$

The mapping of half-lines N and N' into segment M'''M''' of line M is carried out by means of functions $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$, described by the system of Eqs. (28).

The functions $u_5(\rho_{1l})$ and $u_3(\rho_{1l})$ are identically equal for $\rho_{1l} = \rho^I_{1l}$. The functions $u_6(\rho_{1l})$ and $u_4(\rho_{1l})$ are identically equal for $\rho_{1l} = \rho^{II}_{1l}$.

For an initial deviation of $\rho_{1l} = \rho^{**}_{1l} = \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}$ we have

$$\{u_5(\rho_{1l})\}_{\rho_{1l}=\rho^{**}_{1l}} = \mu_d + \frac{\Delta}{2} - \tau\rho_l.$$

From (28) we determine the value of $u_6(\rho_{1l})$ for $\rho_{1l} = \rho^{**}_{1l}$:

$$1 - \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l} = e^{-\frac{\Delta}{\tau} - \rho_l} e^{-\rho_{2f}} (1 + \rho_{2f}).$$

Whence, if we expand $e^{-\rho_{2f}}$ in a series limited just to two terms, we get

$$\rho_{2f} \approx \left\{1 - \frac{1 - \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}}{e^{-\frac{\Delta}{\tau} - \rho_l}}\right\}^{1/2}.$$

For Equality (21') to hold, $\rho_{2f} = 0$, whence $\{u_6(\rho_{1l})\}_{\rho_{1l}=\rho^{**}_{1l}} = \mu_d + \frac{\Delta}{2} - \tau\rho_l$ and, consequently,

$$\{u_6(\rho_{1l})\}_{\rho_{1l}=\rho^{**}_{1l}} = \{u_5(\rho_{1l})\}_{\rho_{1l}=\rho^{**}_{1l}}.$$

Appendix III

We consider the diagram of the point transformations for $\mu_d > \mu_{d\,cr\,II}$, i. e., when the parameter ratio is defined by Eq. (21'')(Fig. 7 a). Then, for $\rho_{1l} = 1$ and $\rho_l < \frac{2\mu_d}{\tau}$, we have

$$\mu_d + \frac{\Delta}{2} < \{u_3(\rho_{1l})\}_{\rho_{1l}=1} < \{u_4(\rho_{1l})\}_{\rho_{1l}=1} = \infty.$$

For $\rho_{1l} = \rho^I_{1l} = \rho_l e^{\frac{2\mu_d}{\tau}}$ and the same conditions, we find

$$\{u_3(\rho_{1l})\}_{\rho_{1l}=\rho^{I}_{1l}} = \mu_d + \frac{\Delta}{2}.$$

The function $u_4(\rho_{1l})$ attains its limiting value, equal to $\mu_d + \frac{\Delta}{2}$, for

$$\rho_{1l} = \rho^{II}_{1l} = 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l}(1+\rho_l).$$

If Inequality (21") holds,

$$1 - e^{-\frac{\Delta}{\tau} - \rho_l} < \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l} = \rho^{**}_{1l}$$

or

$$1 - e^{-\frac{\Delta}{\tau} - 2\rho_l} e^{\rho_l} < \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l} = \rho^{**}_{1l}.$$

If the transcendental function e^{ρ_l} is expanded in a series in which only the first two terms are retained, the sign of the inequality is reversed:

$$\rho^{II}_{1l} = 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l}(1+\rho_l) > \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l} = \rho^{**}_{1l}.$$

We now consider the placement of curves $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$, described by Eqs. (28). The functions $u_3(\rho_{1l})$ and $u_5(\rho_{1l})$ are identically equal for $\rho_{1l} = \rho^{I}_{1l}$. The functions $u_4(\rho_{1l})$ and $u_6(\rho_{1l})$ are identically equal for $\rho_{1l} = \rho^{II}_{1l}$.

For $\rho_{1l} = \rho^{**}_{1l} = \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}$ we have

$$\{u_6(\rho_{1l})\}_{\rho_{1l}=\rho^{**}_{1l}} \approx \mu_d + \frac{\Delta}{2} - \tau\rho_l + \tau\sqrt{1 - \frac{1 - \rho_l e^{\frac{2\mu_d}{\tau} - \rho_l}}{e^{-\frac{\Delta}{\tau} - \rho_l}}},$$

$$\{u_5(\rho_{1l})\}_{\rho_{1l}=\rho^{**}_{1l}} = \mu_d + \frac{\Delta}{2} - \tau\rho_l.$$

Whence we conclude that, if Condition (21") is satisfied,

$$\{u_6(\rho_{1l})\}_{\rho_{1l}=\rho^{**}_{1l}} > \{u_5(\rho_{1l})\}_{\rho_{1l}=\rho^{**}_{1l}}.$$

Appendix IV

In analogous fashion we consider the diagrams of the point transformations when the parameter ratio is defined by Eq. (10) (Fig. 7b).

We analyze the placement of the curves $u_3(\rho_{1l})$ and $u_4(\rho_{1l})$, described by the system of Eqs. (27).

For $\rho_{1l} = 1$, if Equality (10) holds,

$$\mu_d + \frac{\Delta}{2} < \{u_3(\rho_{1l})\}_{\rho_{1l}=1} < \{u_4(\rho_{1l})\}_{\rho_{1l}=1} = \infty.$$

For $\rho_{1l} = \rho_l e^{\frac{2\mu_d}{\tau}} = \rho^{I}_{1l}$ we have

$$\{u_3(\rho_{1l})\}_{\rho_{1l}=\rho_{1l}^{\mathrm{I}}} = \{u_4(\rho_{1l})\}_{\rho_{1l}=\rho_{1l}^{\mathrm{I}}} = \mu_3 + \frac{\Delta}{2}.$$

We consider the functions $u_5(\rho_{1l})$ and $u_6(\rho_{1l})$ which are equal to each other, and to the functions $u_3(\rho_{1l})$ and $u_4(\rho_{1l})$, for $\rho_{1l} = \rho_{1l}^{\mathrm{I}}$.

For $\rho_{1l} = \rho_l$, the function $u_5(\rho_{1l})$ attains its limiting value:

$$\{u_5(\rho_{1l})\}_{\rho_{1l}=\rho_l} = -\left(\mu_d - \frac{\Delta}{2}\right).$$

The function $u_6(\rho_{1l})$ attains the same value, $-\left(\mu_d - \frac{\Delta}{2}\right)$, for the value $\rho_{1l} = \rho_{1l}^{\mathrm{III}}$, which is determined from Eq. (29), from whence it follows that

$$\rho_{1l}^{\mathrm{III}} > 1 - e^{-\frac{\Delta}{\tau} - \rho_l} \quad \text{or} \quad \rho_{1l}^{\mathrm{III}} > 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l} e^{\rho_l}.$$

If the function e^{ρ_l} is expanded in a series and only the first two terms are retained, we obtain

$$\rho_{1l}^{\mathrm{III}} < 1 - e^{-\frac{\Delta}{\tau} - 2\rho_l} (1 + \rho_l)$$

or, if Equality (10) holds,

$$\rho_{1l}^{\mathrm{III}} < \rho_l e^{\frac{2\mu_d}{\tau}} = \rho_{1l}^{\mathrm{II}}.$$

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ANALYSIS OF FREE OSCILLATIONS ABOUT ITS CENTER OF GRAVITY OF A NEUTRAL PLANE WITHOUT DAMPING OF ITS OWN AND WITH A RELAY AUTOPILOT*

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A relay system, the linear part of which is described by the simplest degenerate third-order equation, is considered. The control system's dynamics are analyzed by means of the method of point transformation of surfaces, which enables one to show all the possible modes of movement in it, including the sliding modes.

The analysis yields the equations of surfaces determining the region of attraction (by the initial conditions), equilibrium state, and the stable limiting cycle. These equations may serve as criteria in the choice of the regulator's parameters, under given restrictions, from a coordinate and two of its derivatives.

1. Statement of the Problem

The present work is devoted to the study, by means of the method of point transformation, of the simplest relay system, the linear part of which is described by a degenerate third-order equation. A similar but more general problem was studied by A. A. Andronov and N. N. Bautin [2, 3], B. V. Bulgakov [4], A. M. Letov [5], V. A. Kotel'nikov [6], P. V. Bromberg [7], I. Flugge-Lotz [8], etc. However, results of the study made on the problem being considered cannot be obtained in a particular case from the above papers. In fact in [2] and [3], the authors make use of such generalized coordinates, intrinsic parameters, and dimensionless time, which in our case (M = 0) all become zero.

It should be noted that in the case where the solution is carried out in usual coordinates, it is possible, in principle, to obtain results in the study of a system, the linear part of which is described by a degenerate equation, as a special case of the study of a system described by a complete equation. For this it is necessary to carry out a transition in the limit. However, this problem usually turns out to be more complicated in comparison to an independent study of the system described by the degenerate equation.

The results of [8] cannot be applied to the solution of the given problem not only in view of the above-mentioned complications in applying them, but also because of their incompleteness.

It follows from the papers of the other authors that the system under study is mostly unstable. However, the limits of the system's stability, determined by the initial conditions, have not been found, and there is, therefore, no criterion for the choice of the regulator's parameters. It should be pointed out as well that it does not appear to be possible to find the regions of stability from the initial conditions using the method of the small parameter and the frequency method, while the solution of the given problem by the method of harmonic balance yields principally incorrect results. Thus in the study of the system by the indicated method, we are led to the conclusion that the blind zone in a servomotor leads to oscillations. However, as will be shown further on, the blind zone merely decreases the region of stability determined by the initial conditions.

* The work was carried out under the direction of B. N. Petrov.

Thus it would be expedient to solve the given problem by the method of point transformation which would permit, in general, the derivation of the equations of surfaces determining the region of attraction (by the initial conditions), equilibrium state, and the stable limiting cycle. The latter permits the choice of the regulator parameters in such a way that the system is, under given limiting conditions, efficient.

Solution of the problem is based on [2, 3]. These were the first papers to appear in which the space problem was rigorously solved and regions of stability were established for it in terms of the system's parameters. The theory of the point transformation of surfaces is set forth in them as well.

2. Equations of Motion

Let us examine the free oscillations about its center of gravity, from its angle of course, of a neutral plane having negligible damping of its own, steered by an automatic pilot with a constant speed servomotor.

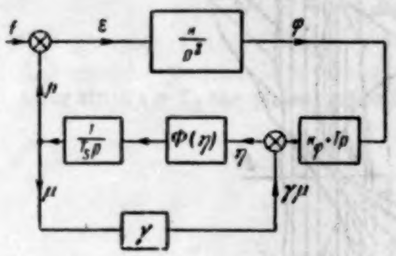

Fig. 1.

The equations of motion of the system under study may be written, after the usual simplifying assumptions, as

$$\ddot{\varphi} = -k\mu, \quad T_S\dot{\mu} = F(\eta), \qquad \eta = k_\varphi \varphi + T\dot{\varphi} - \gamma\mu, \tag{1}$$

where φ is the relative change in the plane's angle of course, μ is the relative change in the angle of rotation of the steering wheel, η is the relative change in the valve's coordinate; k, k_φ, T, γ are the constant coefficients characterizing the parameters of the system; $F(\eta)$ is the relay function which, in general, is not single-valued. A schematic drawing of the SAR is shown in Fig.1.

Let us introduce new variables $\delta = T_S\mu$, $\sigma = \frac{1}{k_\varphi}\eta$, $\psi = -\varphi$ and denote $\frac{T}{k_\varphi} = \beta$, $\frac{\gamma}{T_S k_\varphi} = \frac{1}{a}$, $N = \frac{k}{T_S}$. Then Eq. (1) will take on the form:

$$\ddot{\psi} = N\delta, \quad \dot{\delta} = f(\sigma), \quad \sigma = -\psi - \beta\dot{\psi} - \frac{1}{a}\delta \tag{2}$$

or

$$\dot{z} = N\delta, \quad \dot{\delta} = f(\sigma), \quad \dot{\sigma} = -z - \beta N\delta - \frac{1}{a} f(\sigma),$$

where $z = \dot{\psi}$.

The relay function $f(\sigma)$ is described by the equation

$$f(\sigma) = \begin{cases} 1 & \text{for} \quad \sigma \geqslant \sigma_\varepsilon - \frac{\Delta}{2} \text{ and } f(\sigma_0) = 1, \\ 0 & \text{for} \quad |\sigma| < \sigma_\varepsilon + \frac{\Delta}{2} \text{ and } f(\sigma_0) = 0, \\ -1 & \text{for} \quad \sigma \leqslant -\left(\sigma_\varepsilon - \frac{\Delta}{2}\right) \text{ and } f(\sigma_0) = -1, \end{cases} \tag{3}$$

where σ_ε is the region of insensitivity, Δ is the loop width, σ_0 are the initial conditions at each region of realignment with $t = +0$. (At every change of equations the time origin is reset to zero).

3. The Stabilization of the Plane's Course by Means of an Automatic Pilot with a Relay Servomotor Not Possessing a Blind Zone

We shall consider z, δ, and σ to be the rectangular coordinates of phase space, and examine the solutions of Eqs. (2) under the initial conditions $t = 0$, $z = z_0$, $\delta = \delta_0$ and $\sigma = \sigma_0$. At the same time we assume that $z(t)$, $\delta(t)$ and $\sigma(t)$ are continuous at the points of discontinuity of the relay function $f(\sigma)$.

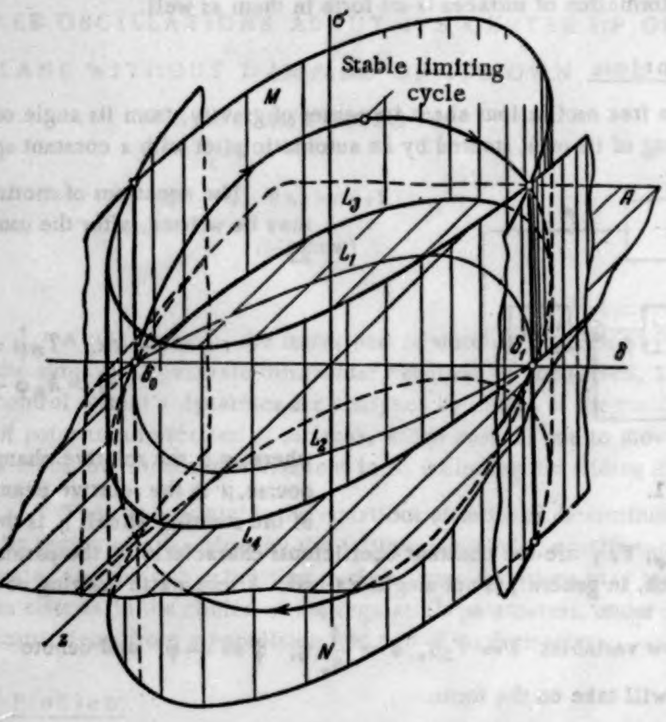

Fig. 2. Phase space with $\sigma_\epsilon = \Delta = 0$.

Plane A ($\sigma = 0$) divides the phase space (z, δ, σ) into two parts (Fig. 2). In the top $f(\sigma) = 1$ and in the bottom $f(\sigma) = -1$. The phase coordinates change in time according to the expression:

$$\delta = f(\sigma)t + \delta_0, \quad z = Nf(\sigma)\frac{t^2}{2} + N\delta_0 t + z_0,$$
$$\sigma = -Nf(\sigma)\frac{t^3}{6} - (\delta_0 + \beta f(\sigma))N\frac{t^2}{2} - \left(z_0 + \beta N\delta_0 + \frac{1}{a}f(\sigma)\right)t + \sigma_0. \tag{4}$$

Projections of phase trajectories on the planes $\sigma = \sigma_0 = \text{const}$ are described by the equation

$$z = \frac{N}{f(\sigma)}\frac{\delta^2}{2} + \left(z_0 - \frac{N}{f(\sigma)}\frac{\delta_0^2}{2}\right), \tag{5}$$

while the projections of the phase trajectories on the vertical planes $z = z_0 = \text{const}$ are the cubic parabolas

$$\sigma = \left[-\frac{N}{6}\delta^2 - \frac{\beta N}{2f(\sigma)}\delta + \frac{z_0}{f(\sigma)} - \frac{N}{2}\delta_0^2 - \frac{1}{a}\right]\delta +$$
$$+ \frac{2}{3}N\delta_0^3 + \frac{\beta N}{2f(\sigma)}\delta_0^2 - \frac{z_0}{f(\sigma)}\delta_0 + \frac{1}{a}\delta_0 + \sigma_0. \tag{6}$$

Thus each phase trajectory appears as a line of intersection of two surfaces, of which one is formed by the movement of the vertical axis according to (5), while the other is determined by Eq. (6) if one substitutes into it $z_0 = z_0(z,\delta)$ in accordance with (5).

Equations (2) determine the point transformation of the plane $\sigma = 0$ into itself. To the extent that phase space is symmetrical with respect to the origin of the coordinates, it suffices to examine the transformation of the left ($\delta < 0$) half-plane $\sigma = 0$ into the right ($\delta > 0$) half-plane $\sigma = 0$. We shall call this transformation S^+. To find S^+, we proceed in the following manner: let the starting point z_0, δ_0 ($t = 0$) be in the plane $\sigma = 0$. Let us use T_1 to designate the smallest value of t at which the phase trajectory, having originated at the point (z_0, δ_0), once again appears on the plane $\sigma = 0$. Then, using the derived value of δ_0, we find z_0 from the last equation in (4):

$$\delta_0 = \delta_0(\zeta, T_1) \equiv -\zeta,$$
$$z_0 = z_0(\zeta, T_1) \equiv -\frac{N}{6}T_1^2 + \frac{N}{2}(\zeta-\beta)T_1 + \beta N\zeta - \frac{1}{a}. \quad (7)$$

After time $t = T_1$ the phase trajectory will cross the plane $\sigma = 0$ at the point z_1, δ_1:

$$\delta_1 = \delta_1(\zeta, T_1) \equiv -\zeta + T_1,$$
$$z_1 = z_1(\zeta, T_1) \equiv \frac{N}{3}T_1^2 - \frac{N}{2}(\zeta+\beta)T_1 + \beta N\zeta - \frac{1}{a}. \quad (8)$$

Equations (7) and (8) yield, in parametric form, the transformation S^+.

We shall begin the study of the point transformation S^+ with the construction of parabola L_1 (Fig. 2), which possesses the property that the phase trajectory crossing it at $\sigma = 0$, $\delta = \delta_0 < 0$, will intersect it again at $\sigma = 0$, after time $t = T_1$. We shall obtain the equation of this parabola by substituting into the second expression of (7) or (8) $T_1 = 2\zeta$:

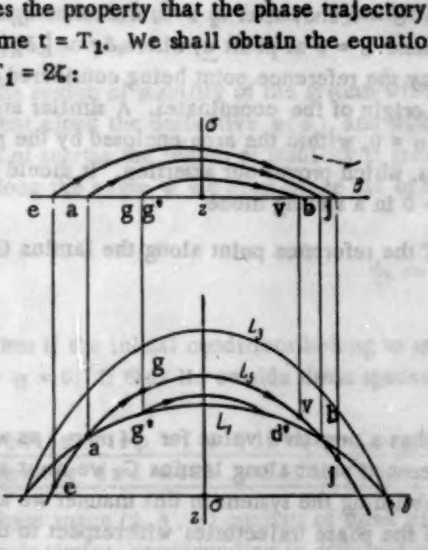

Fig. 3. Determination of the stability of the limiting cycle.

$$z = \frac{1}{3}N\zeta^2 - \frac{1}{a}. \quad (9)$$

For $f(\sigma) = -1$ we shall analogously obtain parabola L_2:

$$z = -\frac{1}{3}N\zeta^2 + \frac{1}{a}. \quad (10)$$

Since for the existence of an ordinary limiting cycle in the system it is necessary to have $\delta_1 = -\delta_0$, such a cycle must pass through the points of intersection of parabolas L_1 and L_2 with the δ-axis:

$$\delta_a = \pm\sqrt{\frac{3}{Na}}. \quad (11)$$

A study of the point transformation shows that in the system, a limiting cycle that passes through points (11) is possible. Its projection in the plane $\sigma = 0$ consists of parabolas L_3 and L_4:

$$z = \pm\frac{N}{2}\delta^2 \mp \frac{3}{2a}. \quad (12)$$

To determine the type of cycle, let us first note that the phase trajectories lying in the arbitrary cylindrical surface, the projection of which on the plane $\sigma = 0$ follows, for example, parabola L_5 (Fig. 3), converge to the trajectory crossing parabola L_1. In fact we shall find from the third equation in (4) the expression for T_1:

$$(\sigma_0 = \sigma_{\text{end}} = 0)$$

$$T_1 = 3\,\frac{-(\delta_0+\beta)\frac{N}{2} + \sqrt{(\delta_0+\beta)^2\frac{N^2}{4} - \frac{2}{3}\beta N^2\delta_0 - \frac{2}{3}\frac{N}{a} - \frac{2}{3}Nz_0}}{N} \tag{13}$$

If $z_0 = z_a < 0$, $\delta_0 = \delta_a < 0$ (Fig. 3), then $T_1 = 2\,|\delta_a|$; if $z_0 = z_{g'} < 0$, $\delta_0 = \delta_{g'} < 0$, then $T_1 = 2\,|\delta_{g'}|$, but if $z_0 = z_g < z_{g'}$, $\delta_0 = \delta_g = \delta_{g'}$, then $T_1 > 2\,|\delta_g|$, and hence $\delta_g > |\delta_g|$. Analogously it can be shown that $\delta_j < |\delta_e|$.

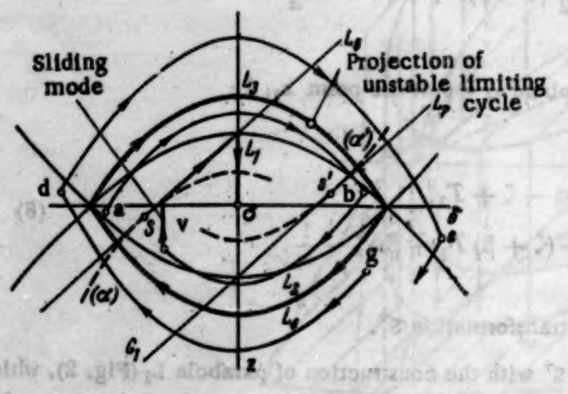

Fig. 4. Projection of phase space on the plane $\sigma = 0$ for the case $\sigma_\varepsilon = \Delta = 0$.

Let us also show that the obtained limiting cycle is an unstable one. For this it suffices to examine the motion of the representative (reference) points which, in the first instant of time, are located on the switching plane $\sigma = 0$. Let us take, for example, point g which lies outside the area enclosed by the projection of the limiting cycle (Fig. 4). After some time $t_1 > 0$ the reference point travelling along the phase trajectory, situated below the plane $\sigma = 0$, will arrive once again on this plane at point d. In this case, in accordance with the property just examined, the inequality $\delta_e > |\delta_d|$ will hold. Thence, the reference point will continue its motion along the trajectory, passing above the plane $\sigma = 0$ and, after $t_2 > 0$, will once again return to plane $\sigma = 0$ at point e, where $\delta_l > |\delta_k|$, etc. In this way the reference point being considered recedes from the origin of the coordinates. A similar argument shows that a reference point which begins its motion on the plane $\sigma = 0$, within the area enclosed by the projection of the limiting cycle, approaches the origin of the coordinates, which proves our assertion. It should also be noted that the system enters a state of equilibrium $z = \delta = \sigma = 0$ in a sliding mode.

The sliding mode in the system is reflected in the motion of the reference point along the lamina G_1 of plane $\sigma = 0$ enclosed by the straight lines L_6 and L_7:

$$z = -\beta N\delta \mp \frac{1}{a}. \tag{14}$$

Equation (14) is obtained from the fact that T_1 on lamina G has a negative value for $f(\sigma) = 1$ as well as for $f(\sigma) = -1$. To determine the nature of the motion of the reference point along lamina G, we must assume that the relay switches over with a certain time delay τ. Pre-establishing the system in this manner we arrive at the obvious result that, assuming symmetry in the projections of the phase trajectories with respect to the z-axis, the reference point moves on the lamina G along the straight lines

$$\delta = \delta_0 = \text{const}. \tag{15}$$

Upon reaching either L_6 or L_7, the reference point moves towards the z-axis along these straight lines if $\delta_S < \delta_0 < \delta_{S'}$, or moves along the corresponding phase trajectories if $\delta < \delta_S$ or $\delta_0 > \delta_{S'}$. Here $\delta_S = -B$ and $\delta_{S'} = +B$ are the abscissas of the points of tangency of the straight lines L_6 and L_7 with the parabolas (α) and (α'), which are determined by Eqs. (5) (points S and S' in Fig. 4).

In [9] it is shown, that in relay systems, the linear part of which is described by an equation of not less than third order, there can appear not only the simple limiting cycles but also ones of any degree of complexity. However, in the problem under consideration, in view of the fact that the equation of the linear part appears as the simplest form of degenerate equation, complex oscillations are absent. This follows from the fact that the reference point, as it was just shown, taken at some instant of time on the plane $\sigma = 0$, outside the contour bounded by parabolas L_3 and L_4, with subsequent motion will continuously recede from the origin of the coordinates (δ incessantly increases). On the other hand, if the reference point on the plane $\sigma = 0$ is within this contour, then subsequent motion will only result in its approaching the origin of the coordinates. For this same reason in the system under consideration there cannot be limiting cycles consisting of sliding and nonsliding motions, the presence of which in high order relay systems has been proved in [10] and [11].

In fact, if the reference point, on leaving the sliding mode lamina G_1, arrives, after one half-cycle of oscillation, on the plane $\sigma = 0$, outside the region enclosed by the contour formed by L_3 and L_4, then the system further on will break into runaway oscillations. In the opposite case it will settle into a state of equilibrium (in a slipping mode).

Thus the system being studied is mostly unstable. The region of stability is, by the initial condition for G_2, enclosed in the space between two cylindrical surfaces M and N, shown in Fig. 2, and bounded above and below by the surfaces whose equations, in parametric form, are

$$\sigma = \left[\frac{N}{6}\delta_k^2 + \frac{\beta N}{2f(\sigma)}\delta_k - \frac{z}{f(\sigma)} + \frac{N}{2}\delta^2 + \frac{1}{a}\right]\delta_k - \frac{2}{3}N\delta^3 - \frac{\beta N}{2f(\sigma)}\delta^2 + \frac{z}{f(\sigma)}\delta - \frac{1}{a};$$
$$\delta_k = f(\sigma)\sqrt{\frac{1}{2}\delta^2 - \frac{z}{Nf(\sigma)} + \frac{3}{2aN}}. \tag{16}$$

Moreover, the system will be stable if the initial conditions correspond to the space G_3 from which the reference point will arrive on lamina G_1 under the conditions $|\delta| < \max(\beta, |\delta^*|)$, where δ^* is the point of intersection of L_4 and L_6.

The region of stability of the system widens with increase of the stable feedback factor $1/a$, the reaction coefficient along the derivative $\dot{\psi}(\beta)$, and with decrease of the servomotor's reset time. For objects with a large moment of inertia the region of stability is greater than for low inertia objects. To determine the region of stability along the angle ψ we must make use of the formula

$$\psi_0 = -\sigma_0 - \beta z_0 - \frac{1}{a}\delta_0. \tag{17}$$

Thus if the initial conditions belong to spaces G_2 and G_3 then the system will arrive at the stable condition $z = \delta = \sigma = 0$; if they lie outside these spaces, the system will break into runaway oscillations.

4. Stabilization of a Plane's Course by Means of an Automatic Pilot with a Relay Servomotor and with a Blind Zone

Phase space (z, δ, σ) consists of three parts: I) $(f(\sigma) = 1)$, II) $(f(\sigma) = 0)$ and III) $(f(\sigma) = -1)$. A joining of trajectories, corresponding to different parts of phase space, occurs on the planes $A(\sigma = \sigma_\epsilon)$ and $B(\sigma = -\sigma_\epsilon)$. In parts I and III the phase coordinates change in time according to (4), the projections of the phase trajectories on the planes $\sigma = \sigma_0 = \text{const}$ are determined by Eq. (5), and on the planes $z = z_0 = \text{const}$ by Eq. (6). In part II the phase trajectories lie in the vertical planes $\delta = \delta_0 = \text{const}$ and are determined by the equation:

$$\sigma = -\frac{1}{N\delta_0}\left[\frac{z^2 - z_0^2}{2} + \beta N\delta_0(z - z_0)\right] + \sigma_0. \tag{18}$$

In view of the symmetry of phase space, shown in Fig. 5, we shall examine the point transformation of plane A not into itself, but the transformation of its left half ($\delta < 0$) into the right half ($\delta > 0$) of plane B.

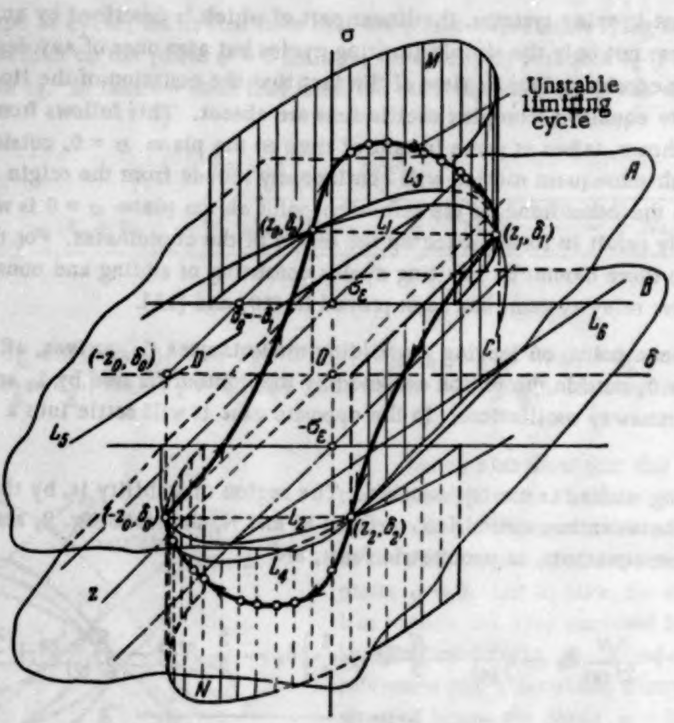

Fig. 5. Phase space with $\sigma_\epsilon \neq 0$, $\Delta = 0$.

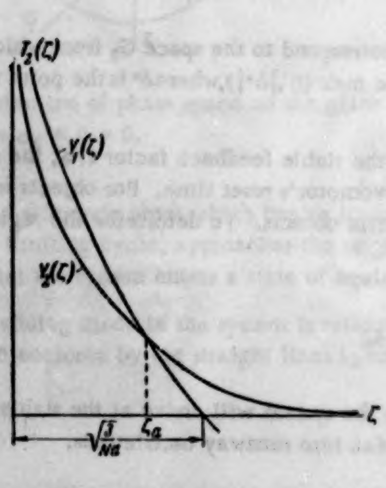

Fig. 6. Diagram of point transformation for the case $\sigma_\epsilon \neq 0$, $\Delta = 0$.

Let us call the transfer of the reference point (z_0, δ_0) from the left half-plane of A to its right half-plane, transformation S^+. Expressions (7) and (8) give the transformation S^+ in parametric form.

The transfer of point (z_1, δ_1) from plane A to plane B with $\delta > 0$ we shall call transformation E^+. We shall define the transformation E^+ by means of the parameter $T_2 > 0$, which is the time required for the point to make the indicated transition. We call the coordinates of the image point on plane B ($\delta > 0$) z_2, δ_2, and write the transformation E^+ in the form

$$\begin{aligned} \delta_2 &= \delta_2(\zeta,\ T_1,\ T_2) \equiv \delta_1, \\ z_2 &= z_2(\zeta,\ T_1,\ T_2) \equiv N\delta_1 T_2 + z_1. \end{aligned} \tag{19}$$

For a simple limiting cycle to exist in the system it is necessary to have $\delta_2 = -\delta_0$, and $z_2 = -z_0$. From the first condition we find that

$$T_1 = 2\zeta, \tag{20}$$

while from the second we have

$$T_2 = \frac{2}{Na\zeta} - \frac{2}{3}\zeta = Y_1(\zeta). \tag{21}$$

Let us now find T_2 from the condition that the reference point (z_1, δ_1), having begun its motion in plane A will, after time T_2, arrive on plane B (z_2, δ_2). For this we can make use of the third equation in (4), after setting $f(\sigma) = 0$ in it. As a result, we shall obtain

$$T_2 = \frac{-\left(\frac{1}{3}N\zeta^2 + \beta N\zeta - \frac{1}{a}\right) + \sqrt{\left(\frac{1}{3}N\zeta^2 + \beta N\zeta - \frac{1}{a}\right)^2 + 4N\zeta\sigma_\varepsilon}}{N\zeta} = Y_2(\zeta). \tag{22}$$

Comparing T_2 obtained from Eqs. (21) and (22), we can find the δ-coordinate of the amplitude of the limiting cycle:

$$\zeta_a = \delta_a = \sqrt{\frac{3}{N}\left(\frac{1}{a} - \frac{\sigma_\varepsilon}{\beta}\right)}. \tag{23}$$

The relative position of curves $Y_1(\zeta)$ and $Y_2(\zeta)$ is shown in Fig. 6. Examination of these curves, taking into account the results of the analysis for the case $\sigma_\epsilon = 0$, shows that there is only one unstable limiting cycle in the system. The functions $Y_1(\zeta)$ and $Y_2(\zeta)$ may have only two common points; $\zeta = 0$ and $\zeta = \delta_a$. The limiting cycle (Fig. 5) goes from the point ($z_0, \delta_0 = \delta_a$) along the cylindrical surface M, then a switch over $f(\sigma)$ from the value 1 to the value 0 occurs on plane A and the image point from then on follows the vertical plane C ($\delta = -\delta_0$ = const) up to plane B. The second half of the cycle is symmetrical with the one just described with respect to the origin of the coordinates.

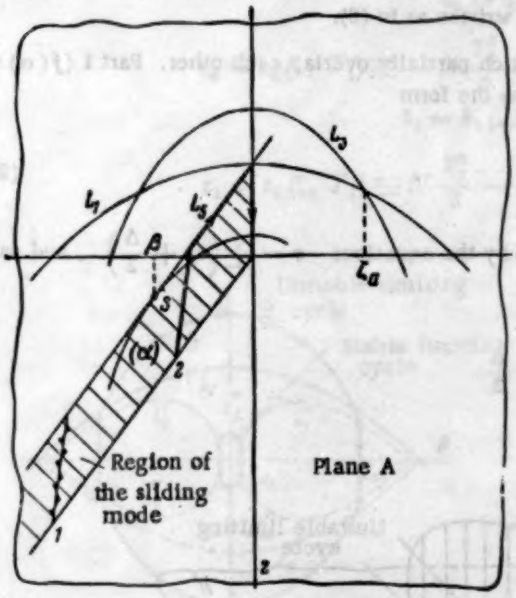

Fig. 7. Determination of the region of the sliding mode on plane A.

A sliding mode, which travels either along the switching plane A with $\delta < 0$ and with $-\beta N\delta > z > -\beta N\delta - 1/a$, or along plane B with $\delta > 0$ and $-\beta N\delta < z < -\beta N\delta + 1/a$, is possible in the system. Figure 7 shows the sliding mode in the system for the case when it travels along plane A. As can be seen from the diagram, the distribution of the regions of the sliding mode when $\sigma_\epsilon \neq 0$, is significantly different from the distribution for the case $\sigma_\epsilon = 0$. Moreover, the predetermination of the sliding mode at the expense of the delay in the relay shows that the reference point moves along inclined straight lines and not along vertical ones, as when $\sigma_\epsilon = 0$. The slope of the straight lines depends on the magnitude of the delay. If the reference point in the sliding mode reaches the straight line L_s with $|\delta| < \beta$, then, analogously to the case when $\sigma_\epsilon = 0$, it moves along the boundary L_s to the z-axis and enters a state of equilibrium($z = 0$, $\delta = 0$, $\sigma = \sigma_\epsilon$).

As in the case $\sigma_\epsilon = 0$, complex cycles and limiting cycles with partially sliding motion cannot exist in the system.

In this way, the system has remained mainly unstable for the case $\sigma_\epsilon \neq 0$ too. The blind zone narrows the region of stability in G_2, which is now enclosed in the space between two cylindrical surfaces M and N, two vertical planes C and D (Fig. 5), and above and below is bounded by the surfaces, the equations of which, in parametric form, are

$$\sigma = \left[\frac{N}{6}\delta_k^2 + \frac{\beta N}{2f(\sigma)}\delta_k - \frac{z}{f(\sigma)} + \frac{N}{2}\delta^2 + \frac{1}{a}\right]\delta_k - \frac{2}{3}N\delta^3 - \frac{\beta N}{2f(\sigma)}\delta^2 +$$
$$+ \frac{z}{f(\sigma)}\delta - \frac{1}{a}\delta + \sigma_\varepsilon f(\sigma),$$
$$\delta_k = f(\sigma)\sqrt{\frac{1}{2}\delta^2 - \frac{z}{Nf(\sigma)} - \frac{2}{3aN}} \quad \text{for } |\delta| < \delta_a, \tag{24}$$
$$\delta_k = \delta_a f(\sigma) \quad \text{for } |\delta| > \delta_a.$$

Apart from this, the region of stability widens at the expense of the space from which the reference point arrives in the sliding mode regions of planes A and B with $|\delta| < \max(\beta, \delta_a)$. Oscillations cannot arise in the system, as in the case when $\sigma_\epsilon = 0$.

Comparing the system being studied with the system analyzed in [2, 3], we see that the plane's own damping changes the structure of the phase space to such an extent that the system becomes mainly stable. Artificial damping cannot render the system mainly stable. It merely increases the region of stability by a small amount.

5. Stabilization of Plane's Course by an Automatic Pilot with a Relay Servomotor Having a Blind Zone and a Loop in Its Static Characteristic

We shall examine the stabilization of the course of a neutral plane, having negligible damping of its own, by means of an automatic pilot with a relay servomotor, having a blind zone and a loop in its static characteristic. The equation of the nonlinear function $f(\sigma)$ is, in this case, written as in (3).

Phase space (z, δ, σ) consists of three parts (Fig. 8) which partially overlap each other. Part I ($f(\sigma) = 1$) is bounded from below by plane A", the equation of which has the form

$$\sigma = \sigma_\epsilon - \frac{\Delta}{2}. \tag{25}$$

Part II ($f = 0$) is enclosed by planes A' and B' defined by the equations $\sigma = \pm\left(\sigma_\epsilon + \frac{\Delta}{2}\right)$, and part III ($f(\sigma) = -1$) lies below plane B", defined by the equation

$$\sigma = -\sigma_\epsilon + \frac{\Delta}{2}.$$

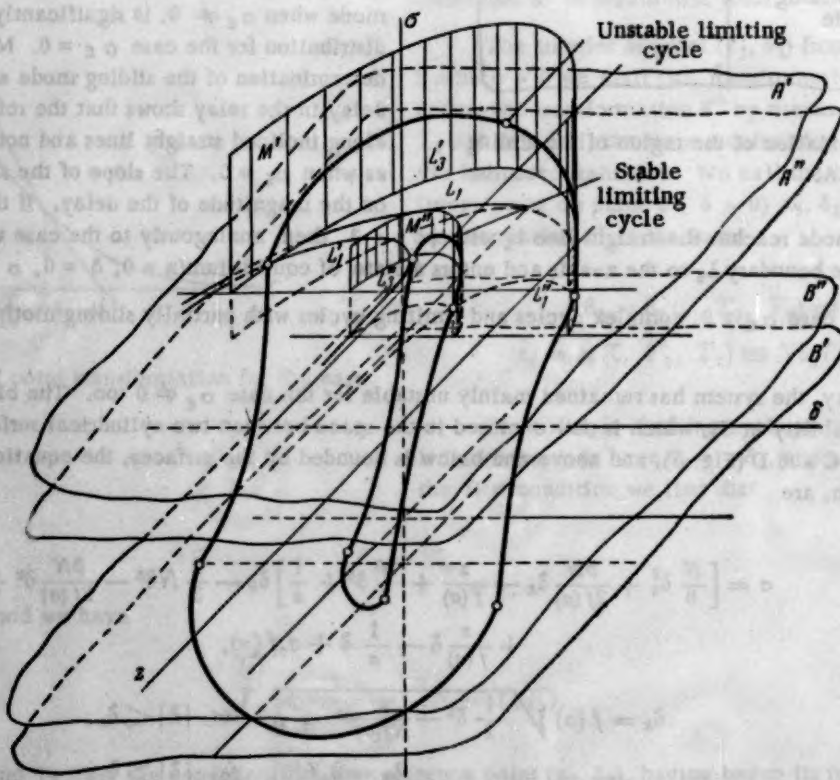

Fig. 8. Phase space with $\sigma_\epsilon \neq 0$ and $\Delta \neq 0$.

The phase coordinates change in time in accordance with (4). Trajectories in parts I and III follow the vertical cylindrical surfaces which form the parabolas, defined in (5), at their intersections with horizontal planes. Projections of the phase trajectories on the planes $z = z_0$ are determined from Eq. (6). Part II is filled with parts of phase trajectories which are situated in the planes $\delta_a = \delta_0 = \text{const}$. Their equations have the form of (18). The joining of trajectories having the same direction takes place on the planes A" and B' when $\delta > 0$ and on the planes B" and A' when $\delta < 0$. The sliding mode in the system is reflected in the motion of the representative (reference) point between planes A' and A" or B' and B".

The transformations S^+, characterizing the transition of the reference point (z_0, δ_0) from plane A' ($\delta < 0$) into the point (z_1, δ_1) on plane A" ($\delta > 0$), has the form

$$
\begin{aligned}
\delta_0 &= \delta_0(\zeta,\ T_1) \equiv -\zeta, \\
z_0 &= z_0(\zeta,\ T_1) \equiv -N\frac{T_1^2}{6} + \frac{N}{2}(\zeta-\beta)T_1 + \beta N\zeta - \frac{1}{a} + \frac{\Delta}{T_1}, \\
\delta_1 &= \delta_1(\zeta,\ T_1) \equiv -\zeta + T_1, \\
z_1 &= z_1(\zeta,\ T_1) \equiv N\frac{T_1^2}{3} - \frac{N}{2}(\zeta+\beta)T_1 + \beta N\zeta - \frac{1}{a} + \frac{\Delta}{T_1},
\end{aligned}
\tag{26}
$$

where T_1 is the time of the indicated transition. The transformation E^+, which characterizes the transition of the reference point (z_1, δ_1) from the plane A" ($\delta > 0$) into the point (z_2, δ_2) on plane B', is written in the form

$$
\begin{aligned}
\delta_2 &= \delta_2(\zeta,\ T_1,\ T_2) \equiv \delta_1, \\
z_2 &= z_2(\zeta,\ T_1,\ T_2) \equiv N\delta_1 T_2 + z_1,
\end{aligned}
\tag{27}
$$

where T_2 is the time required for the reference point to cover the distance between the planes A" and B'.

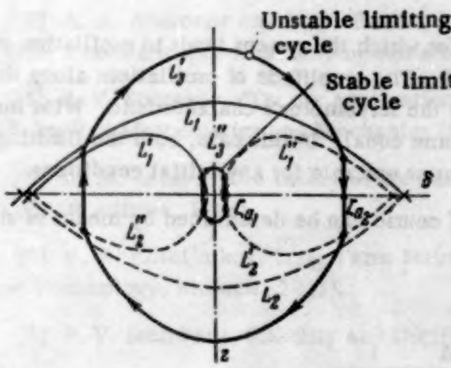

Fig. 9. Projection of phase space on the plane $\sigma = 0$ for the case $\sigma_\epsilon \neq 0$, $\Delta \neq 0$.

For the existence of a limiting cycle in the system it is necessary to have

$$\delta_2 = -\delta_0,\ z_2 = -z_0. \tag{28}$$

From the first condition we find that $T_1 = 2\zeta$. With $\Delta = 0$, parabolas L_1 and L_2 were the geometric positions of points satisfying this equality. Here these parabolas break up into four lines: curve L_1', lying in the left half-plane of A', curve L_1'', lying in the right half-plane of A", curve L_2', lying in the right half-plane of B', and curve L_2'', lying in the left half-plane of B" (Fig. 9). These curves are defined by the equations

$$z = \pm\left(\frac{N}{3}\delta^2 - \frac{1}{a} \mp \frac{\Delta}{2\delta}\right). \tag{29}$$

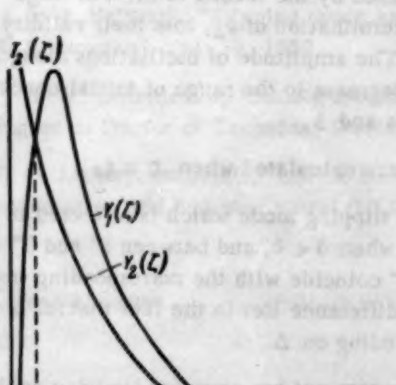

Fig. 10. Diagram of point transformation for the case $\sigma_\epsilon \neq 0$, $\Delta \neq 0$.

From the second equation in (28) we find T_2, taking into account that $T_1 = 2\zeta$,

$$T_2 = \frac{2}{Na\zeta} - \frac{2}{3}\zeta - \frac{\Delta}{N\zeta^2} = Y_1(\zeta). \tag{30}$$

From the third equation of (4), setting $f(\sigma) = 0$ in it, we find T_2, the time necessary for the reference point to cross from plane A" to the plane B':

$$T_2 = \frac{\sqrt{\left(\frac{N}{3}\zeta^2 - \frac{1}{a} + \beta N\zeta + \frac{\Delta}{2\zeta}\right)^2 + 4N\zeta\sigma_\varepsilon} - \left(\frac{N}{3}\zeta^2 - \frac{1}{a} + \beta N\zeta + \frac{\Delta}{2\zeta}\right)}{N\zeta} = Y_2(\zeta) \tag{31}$$

The curves $Y_1(\zeta)$ and $Y_2(\zeta)$, shown in Fig. 10, have two points of intersection, ζ_{a_1} and ζ_{a_2}, which determine two limiting cycles. An analysis of these curves, with the results of the studies set forth in the two preceding sections taken into account, shows that the limiting cycle with the small amplitude ζ_{a_1} is stable while the one having the large amplitude ζ_{a_2} is unstable, i. e., that the system, as before, with certain values of initial inclination, goes into runaway oscillations. Solving Eq. (30) analytically, we arrive at the equation

$$\frac{1}{3}\beta N\zeta^3 - \left(\frac{\beta}{a} - \sigma_\varepsilon\right)\zeta + \frac{\Delta}{2}\beta = 0. \tag{32}$$

The roots of this equation, for small values of Δ, can be calculated approximately as in (12):

$$\zeta_{a_1} \approx \frac{\frac{\Delta}{2}}{\frac{1}{a} - \frac{1}{\beta}\sigma_\varepsilon}, \qquad \zeta_{a_2} \approx \sqrt{\frac{3}{N}\left(\frac{1}{a} - \frac{\sigma_\varepsilon}{\beta}\right)}. \tag{33}$$

Thus, for small values of Δ, the range of initial conditions for which the system tends to oscillation practically coincides with the small region of stability for the case $\Delta = 0$. The amplitude of oscillations along the δ-coordinate is directly proportional to the magnitude of the loop in the servomotor's characteristic. With increase in Δ the roots ζ_{a_1} and ζ_{a_2} converge and for some value of Δ become equal. In this case, both the limiting cycles merge into one semi-stable limiting cycle, while the system becomes unstable for any initial conditions.

The amplitude of the plane's oscillations along the angle of course can be determined by means of simple transformation

$$\psi_a \approx \frac{\beta N}{12}\frac{\Delta^2}{\left(\frac{1}{a} - \frac{\sigma_\varepsilon}{\beta}\right)^2} + \frac{\Delta}{2}\frac{1}{\frac{\beta}{a\sigma_\varepsilon} - 1}. \tag{34}$$

An analysis of Expression (34) shows that ψ_a is mainly determined by the second term. For large N, when the first terms becomes large, Eqs. (33), which were used in the determination of ψ_a, lose their validity. In this case ζ_{a_1} and ζ_{a_2} must be determined from graphical construction. The amplitude of oscillations along angle ψ decreases with increase in σ_ε (however, this is accompanied by a decrease in the range of initial conditions for which the system is stable) and with increase in the coefficients $1/a$ and β.

The period of oscillation is $T = 2(T_1 + T_2)$, where T_1 and T_2 are calculated when $\zeta = \zeta_{a_1}$.

As was pointed out earlier on, there can exist in the system a slipping mode which is reflected in phase space by the motion of a reference point between planes A' and A" when $\delta < 0$, and between B' and B" when $\delta > 0$. The regions of the sliding mode on A' and A" and on B', B" coincide with the corresponding regions on planes A and B for the case when $\sigma_\varepsilon \neq 0$, and $\Delta = 0$ (Fig. 7). The difference lies in the fact that for $\Delta \neq 0$ the amplitude and frequency of oscillation assume limiting values depending on Δ.

A study of the phase space shows that there can exist in the system neither complex limiting cycles nor limiting cycles composed of slipping and nonslipping modes.

In this way, for $\Delta \neq 0$, the system oscillates for all values of its parameters. The amplitude of oscillation is determined by Eqs. (34). However, the system does not tend to break into oscillation for all initial conditions. As in the case of $\Delta = 0$, the system breaks into runaway oscillations for certain values of the initial conditions.

The range of initial inclinations for which the system tends to oscillate approximately coincides with the region of attraction of the equilibrium position when $\sigma_\epsilon \neq 0$, $\Delta = 0$.

SUMMARY

1. The neutral plane, having negligible damping of its own, controlled by an automatic pilot with a constant speed steering servomotor, as a dynamic system is, for the case $\Delta = 0$, mainly unstable.

2. The range of initial conditions for which the system tends to the equilibrium position ($\Delta = 0$) or to oscillation ($\Delta \neq 0$) increases with increase in the object's inertia, steady feedback factor, coefficient of reaction along the derivative of the controlled coordinate, and with decrease in the servomotor's switchover time.

3. The blind zone and the loop in the servomotor's characteristic decrease the range of initial conditions for which the system tends to an equilibrium position ($\Delta = 0$) or to oscillation ($\Delta \neq 0$).

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ELECTROMECHANICAL CALCULATING DEVICE

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The operating principles and description of the author's proposed electromechanical calculating device (ECD) are given. Some results are given of experiments conducted on the simplest type of ECD, which may be used to plot Mikhailov curves, inverse amplitude-phase characteristics of single-loop systems, and limits of D-splitting as a function of the amplification factor, to make functional transformations of speed feedback, and to find real roots of algebraic equations.

In the analysis of the stability and quality of automatic control systems, and especially in the utilization of frequency methods to this end, there often arises the need to calculate complex expressions. Such calculations may be carried out on electronic miniature-scale setups and on digital computers, as well as on simpler electronic and electromechanical units, with an accuracy acceptable in engineering calculations.

For this one may make use of, with some modifications, electronic and electromechanical isographs — instruments for calculating the roots of algebraic equations [1, 2]. The simplest among these, although not the fastest, are the electromechanical calculating devices.

1. Operating Principle and Description of the Simplest Type of an ECD

The principle of operation of the ECD is based on the property of synchronous generators, whereby they develop a voltage, depending in magnitude on the frequency (speed of rotation) and the excitation current, as well as on the frequency-dependent properties of electrical reactive elements (capacitors and inductors) which are supplied with power by synchronous generators.

Figure 1 shows the main single line circuit diagram of connections in the transformation section and one channel of the ECD. In the lower part of the diagram are shown three synchronous generators SG_1, SG_2, SG_3, which are brought into motion by the dc motor DPT, having a wide range of control over the speed of rotation. An emf, proportional in magnitude to the frequency (speed of rotation), is applied to the stator winding of synchronous generator SG_1 which has fixed magnets. A voltage $U_1 = a_1\omega$, proportional to the first power of the frequency is tapped off the potentiometer R_1, which is connected through a rectifier to the stator winding of SG_1.

A voltage proportional to the square of the frequency is obtained by means of a quadratic function converter [4, 5], consisting of a capacitor C_1 and resistor R_2. In fact, if the capacitive reactance is much larger than the resistance over the whole range of frequency variation, then the current flowing in the circuit is $I \approx E_1/x_{C_1} = k_0 C_1 \omega^2$, where $E_1 = k_0(\omega)$ is the emf of the first generator SG_1.

The voltage drop across the resistance R_2 is proportional to the square of the frequency:

$$U_2 = IR_2 = k_0 C_1 R_2 \omega^2 = a_2\omega^2. \tag{1}$$

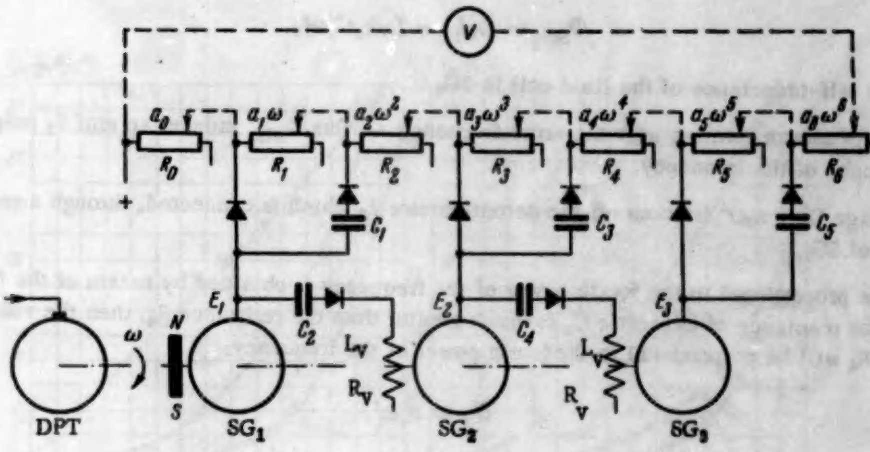
V
a_0
$a_1\omega$
$a_2\omega^2$
$a_3\omega^3$
$a_4\omega^4$
$a_5\omega^5$
$a_6\omega^6$
R_0
R_1
R_2
R_3
R_4
R_5
R_6
C_1
C_2
C_3
C_4
C_5
E_1
E_2
E_3
L_V
R_V
N
S
ω
DPT
SG_1
SG_2
SG_3

Fig. 1.

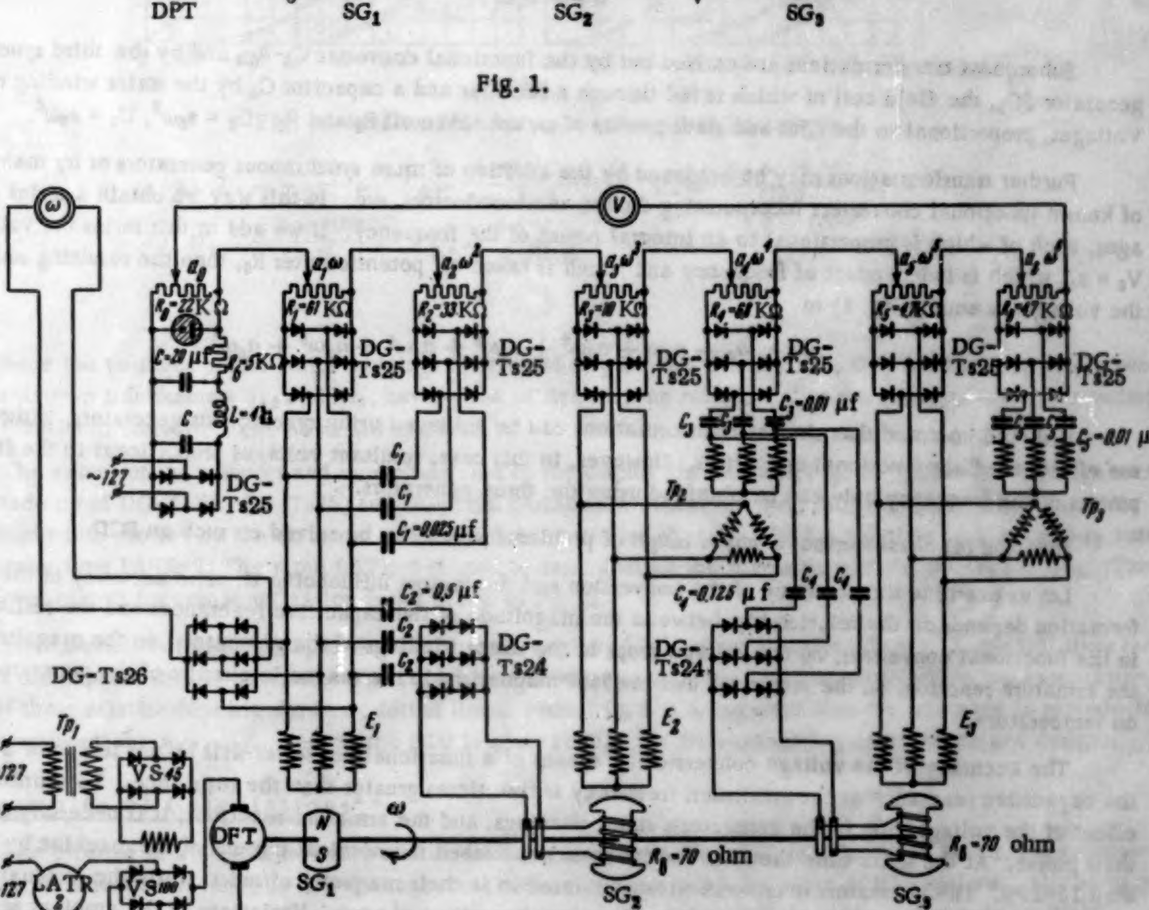
ω
V
DG-Ts25
DG-Ts24
DG-Ts26
$C_1 = 0.025\ \mu f$
$C_2 = 0.5\ \mu f$
$C_4 = 0.125\ \mu f$
$C_5 = 0.01\ \mu f$
~127
VS45
VS100
DFT
LATR
SG_1
SG_2
SG_3
R_1 = 70 ohm
E_1
E_2
E_3

Fig. 2.

For voltages proportional to the cube of the frequency, he second synchronous generator is used as well. Its field coil is fed by the stator winding of SG_1 through a rectifier and capacitance C_2. Since in this circuit too the capacitive reactance is much larger than the resistance R_V of the field coil of SG_2, the excitation current flowing through it is proportional to the square of the frequency. If the magnetic system of SG_2 is not saturated, then this current produces a magnetic flux Φ_{SG_2} proportional to the square of the frequency:

$$\Phi_{SG_2} = L_V I_1 = L_V k_0 C_2 \omega^2, \tag{2}$$

where L_V is the self-inductance of the field coil in SG_2.

With the generator rotating with the same frequency ω, flux Φ_{SG_2} induces an emf E_2 proportional, in magnitude, to the cube of the frequency.

The voltage $U_3 = a_3\omega^3$ is taken off the potentiometer R_3 which is connected, through a rectifier, to the stator winding of SG_2.

A voltage proportional to the fourth power of the frequency is obtained by means of the functional converter C_3-R_4. If the reactance of capacitor C_3 is much greater than the resistance R_4, then the voltage drop across potentiometer R_4 will be proportional to the fourth power of the frequency:

$$U_4 = a_4\omega^4. \tag{3}$$

Subsequent transformations are carried out by the functional converter C_5-R_6, and by the third synchronous generator SG_3, the field coil of which is fed through a rectifier and a capacitor C_4 by the stator winding of SG_2. Voltages, proportional to the fifth and sixth powers of ω, are taken off R_5 and R_6: $U_5 = a_5\omega^5$, $U_6 = a_6\omega^6$.

Further transformations may be achieved by the addition of more synchronous generators or by making use of known functional converters incorporating diodes, semiconductors, etc. In this way we obtain a series of voltages, each of which is proportional to an integral power of the frequency. If we add to this series the voltage $V_0 = a_0$, which is independent of frequency and which is taken off potentiometer R_0, then the resulting sum of the voltages is equal (Fig. 1) to

$$U_\Sigma = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4 + a_5\omega^5 + a_6\omega^6. \tag{4}$$

It should be noted that similar transformations can be achieved using synchronous generators, without the use of intermediate functional converters. However, in this case, resultant voltages proportional to the first three powers of the frequency only can be obtained using the three generators.

Equation (4) characterizes a whole range of problems which can be solved on such an ECD.

Let us examine the accuracy of the conversion and the factors influencing it. The accuracy of the transformation depends on the relationship between the magnitude of the capacitive reactances and the resistances in the functional converters, on the voltage drops in the stator windings of the generators, on the magnitude of the armature reaction, on the saturation and residual magnetism in the magnetic circuits of the generators, and on temperature.

The accuracy of the voltage conversion by means of a functional converter will be not less than 2%, if the capacitive reactance at the maximum frequency is five times greater than the resistance. To eliminate the effect of the voltage drop in the generator's stator windings, and the armature reactions, it is necessary to raise their power. At the same time the load current must not exceed the nominal current of the machine by more than 10-20%. The generators must work without saturation in their magnetic circuits, while the residual magnetism must be eliminated every time before the commencement of work. Variations in the ambient temperature practically have no effect on the accuracy of the transformations since in all the circuits the main impedance is the capacitive one. It should be noted that the apparent power of the synchronous generator is determined by its maximum voltage E and the permissible value of the excitation current in the following generator. The currents flowing in the load potentiometers and the functional converters (apart from those which are set up in the excitation circuits), may be made as small as is desired.

2. Experimental Assembly

Figure 2 shows the main electrical circuit diagram of the transformation section and one channel of an ECD* model. The synchronous generators used were: SG_1 — an aeronautical speed-voltage generator type 2UG1-48 (two-pole, three-phase synchronous generator with fixed magnets, power 30 volt-amperes, voltage 75 v at 3000 rpm), SG_2 and SG_3 — three-phase synchronous generators type FK 8,6-8 (eight-pole, 65 volt-amperes, current 1.05 amperes, voltage 36 v at 7500 rpm, frequency 500 cycles).

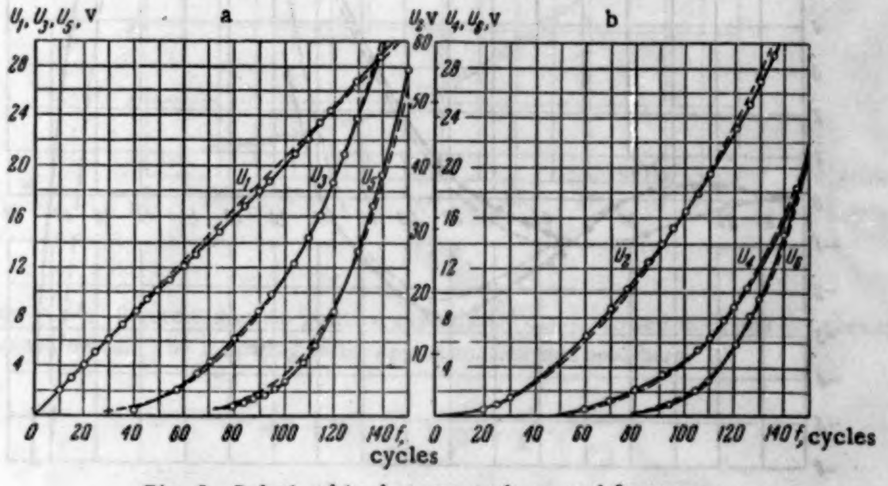

Fig. 3. Relationships between voltage and frequency:

a) $U_1 = \frac{0.203}{2\pi}\omega$, $U_3 = \frac{1.08 \times 10^{-5}}{(2\pi)^3}\omega^3$, $U_5 = \frac{3.4 \times 10^{-10}}{(2\pi)^5}\omega^5$,

b) $U_2 = \frac{3.3 \times 10^{-3}}{(2\pi)^2}\omega^2$, $U_4 = \frac{4.4 \times 10^{-8}}{(2\pi)^4}\omega^4$, $U_6 = \frac{1.9 \times 10^{-13}}{(2\pi)^6}\omega^6$

Since the voltages on the stator windings of SG_2 and SG_3 are relatively small, they are supplied with power through step-up transformers Tr_2 and Tr_3, having one to five step-up ratios, so that the voltages across the resistances of the functional converters can be raised to 20-25 v.

The values of the resistors and capacitors used in the circuit are shown in Fig. 2. Three-phase bridge rectifiers made up of DG-Ts24, DG-Ts25, and DG-Ts26 germanium diodes are used in the system. The generators are brought into motion by the dc motor DPT, which is controlled through a bridge rectifier by a laboratory autotransformer type LATR-2. The same LATR-2 is used to demagnetize the magnetic circuits of SG_2 and SG_3. The switchover circuit for demagnetization is not shown in Fig. 2.

The constant voltage $V_0 = a_0$ is obtained from a regulated full-wave rectifier. In Fig. 3, the full lines represent the experimentally determined relationships between the voltages and frequency, while the approximations of these relationships are shown as dotted lines. From Fig. 3 it is apparent that the accuracy in reproducing the power functions $a_1\omega, \ldots, a_6\omega^6$ on the ECD is quite acceptable for engineering calculations and designs.

3. Examples of Calculations**

The plotting of a Mikhailov curve and of the limits of D-splitting with amplification factor, the calculation of a transcendental function and the real roots of an algebraic equation, serve as illustrations. All the calculations may be carried out either in the true dimensional scale of the variable, or by going over to relative values of the variable and the corresponding transformed equation. The order in which the equation's coefficients are set up and the process of calculation are contained in what follows. Some value of frequency is chosen as

* The model of the ECD was built by N. I. Karmatskii.

** All calculations and drawings were done by R. F. Shepenina and E. A. Borisova.

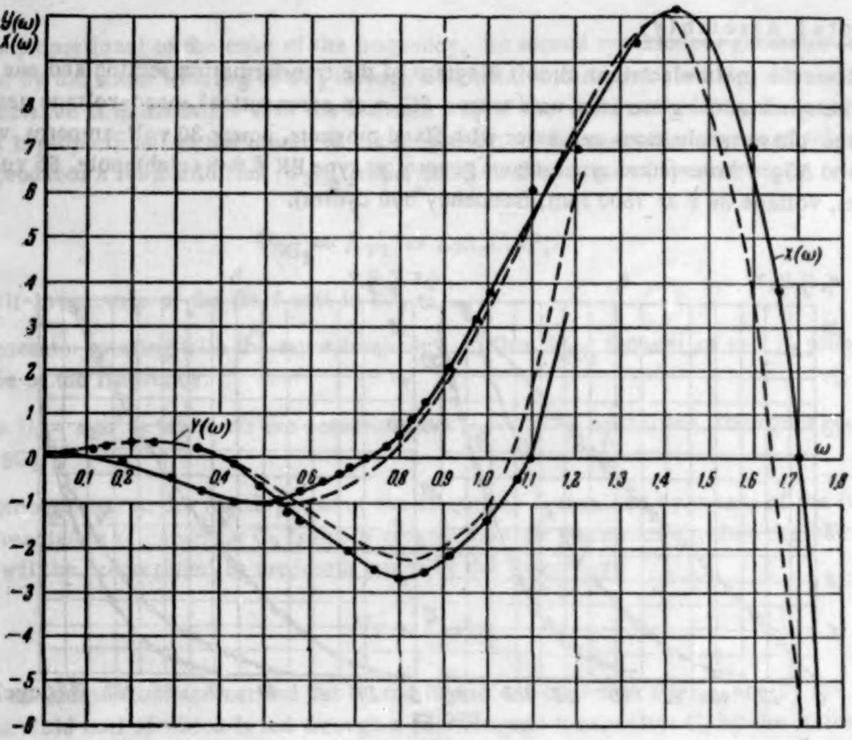

Fig. 4.

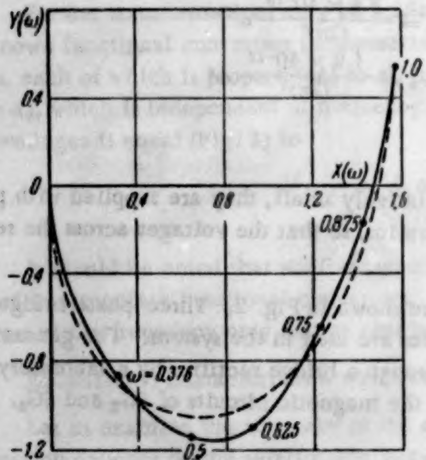

Fig. 5.

a provisory unit. The magnetic circuits of SG_2 and SG_3 are demagnetized, motor DPT is started and the required frequency is established. The coefficients of the equation are set up on potentiometers R_1-R_6 with the help of a VTVM. DPT is stopped and SG_2 and SG_3 are demagnetized. Then all the output terminals of the potentiometers are connected in series and are connected to the voltmeter. The signs of the coefficients are taken into account when connecting up the circuit. Readings are taken from the voltmeter and frequency meter as the motor's rpm is slowly increased. For visual observation, and to obtain a record of the output voltage as a function of frequency, an oscillograph with a memory tube may be used.

Plotting a Mikhailov curve. Figure 4 shows the variation of the real $x(\omega)$ and the imaginary $y(\omega)$ components of a Mikhailov curve as a function of frequency, for the equation

$$p^6 + 2.5\,p^5 + 3.32\,p^4 + 3.725\,p^3 + 1.67\,p^2 + 0.389\,p + 0.025 = 0.$$

The calculated curves are shown dotted while the solid lines give the experimentally determined results.

Plotting the limits of D-splitting with k + 1. Figure 5 shows the graph of the limit of D-splitting with respect to the parameter k + 1 for the equation

$$k + 1 = -\,p^5 - p^4 - 4p^3 - 2.5\,p^2 - 3p.$$

The calculated curve is dotted while the experimental results are given by the solid curve. The range of variation of the relative frequency is shown on the curve.

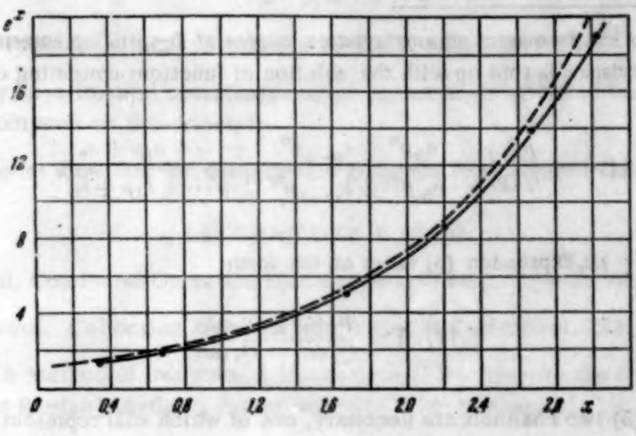

Fig. 6.

Calculation of e^x. Figure 6 shows a positive exponential curve (dotted line) and the corresponding experimentally determined curve. The following series approximation was used here:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\,.$$

Extraction of real roots of an algebraic equation. Figure 7 illustrates the results of the extraction of the real roots of the algebraic equation

$$y = x^5 - 1.2\,x^4 + 0.68\,x^3 - 0.08\,x^2 - 0.9\,x + 0.5$$

by graphical means. The points of intersection of the curve y(x) with the abscissa correspond to the real roots of this equation. The dotted curve is the calculated one while the solid curve is the experimental one.

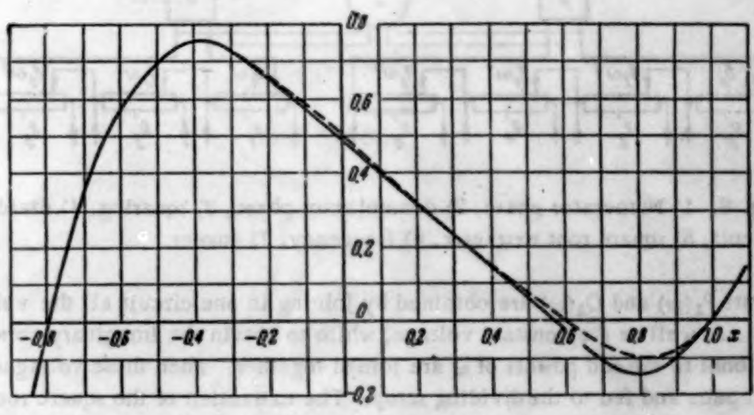

Fig. 7.

As was pointed out earlier on, the ECD or its separate circuits can be applied to the functional transformation of speed feedback, the solution of complicated functions, etc. The range of problems soluble by the ECD can be considerably increased by introducing various nonlinear functional converters (combinations of capacitors and inductors).

4. Complete Schematic of the ECD

The plotting of explicit frequency characteristics, curves of D-splitting interms of two parameters, and amplitude-phase characteristics is tied up with the solution of functions consisting of the ratio of two polynomials of the form

$$A(p) = \frac{P(p)}{Q(p)} = \frac{a_n p^n + a_{n-1} p^{n-1} + \cdots + a_1 p + a_0}{b_m p^m + b_{m-1} p^{m-1} + \cdots + b_1 p + b_0}. \tag{5}$$

Upon substituting $p = j\omega$, Expression (5) takes on the form

$$A(j\omega) = \frac{P_1(\omega) + jP_2(\omega)}{Q_1(\omega) + jQ_2(\omega)}. \tag{6}$$

For the solution of (6) two channels are necessary, one of which will represent the numerator, the other the denominator. Moreover, the forming of the voltages in the second channel (denominator) is done by the same machines which are used for the first one. The complete structural schematic of the ECD is shown in Fig. 8. The voltages from the synchronous generators and the functional converters (not shown in the schematic) are impressed on potentiometers R_1-R_6 and R_1'-R_6'. The upper potentiometers are designated for setting up the numerator, the lower the denominator.

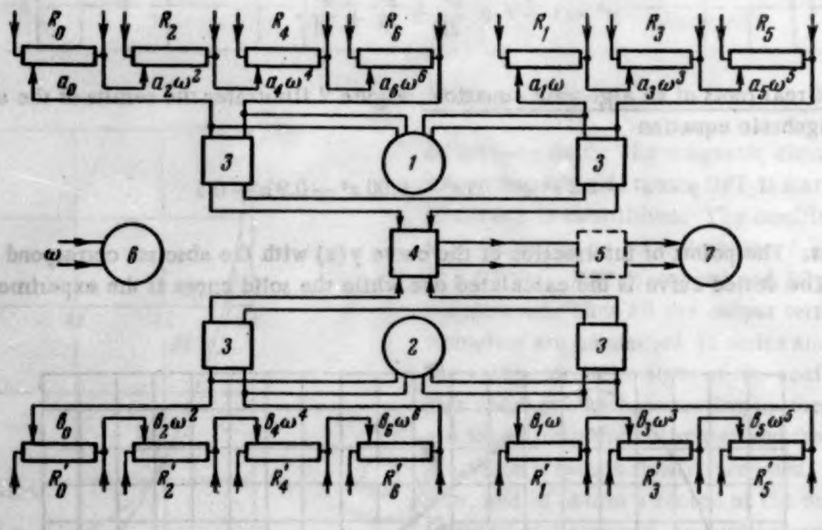

Fig. 8. 1) Numerator phase, 2) denominator phase, 3) squaring, 4) dividing circuit, 5) square root extractor, 6) frequency, 7) answer.

The components $P_1(\omega)$ and $Q_1(\omega)$ are obtained by joining in one circuit all the voltages proportional to the even powers of ω as well as the constant voltage, while to obtain the imaginary components $P_2(\omega)$ and $Q_2(\omega)$ the voltages proportional to the odd powers of ω are joined together. Then these voltages are fed to the squaring circuit, are added in pairs and fed to the dividing setup. The extraction of the square root may be omitted if the instrument is graduated in the units corresponding to the square root of the quantity being measured.

If an oscilloscope or an ordinary voltmeter is used as the measuring instrument then the extraction of the square root cannot be omitted. Ratio meters are used to measure the phases of the numerator and denominator. Division is carried out by means of a ratio meter or any other arrangement.

In this way the ECD, with two channels for setting up voltages, together with the squaring circuits and dividers, enables one to calculate amplitude-phase characteristics, plot frequency characteristics and curves of D-splitting in terms of one or two parameters, and, in general, to calculate the values of rational-fraction functions.

SUMMARY

1. The proposed electromechanical calculating device is sufficiently simple in construction.

2. The preliminary experimental operations carried out on one channel of the ECD confirmed the possibility of constructing a computer on this principle.

3. The cited examples show that the computation accuracy is acceptable for engineering calculations.

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* See English translation.

CIRCULAR TRANSMISSION OF REMOTE CONTROL SIGNALS WITH A COMBINATORIAL METHOD OF CHOICE

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Various methods of choice are investigated for time-separated signals, and the possibility of circular transmission of signals using a combinatorial method of selection is demonstrated. Recommendations are given for the use of interdependent pulse coding together with a "stepping-switch" method of choice, and the effectiveness of such a code is determined.

For the remote control of objects focused on a small number of massive points to be controlled (electric power stations, traction and step-down substations, etc.) by tele-controlling and tele-signalling (TC-TS) devices, the requirement is usually posed that there be guaranteed circular transmission of information signals concerning the objects controlled. Significantly less frequently is the requirement posed for circular transmission of the command signals controlling the objects, and special measures are even taken to guard against the execution of command signals containing more than one order.

The great advantage of circular transmission of information signals is shown with particular sharpness in cases where it is necessary to re-establish quickly the picture of the controlled objects' state after a disruption of the TC-TS device's functioning or during an emergency, and also for the control of connected or assembled objects for which a change of state of one induces a change of state in many others. Thus, for example, a deviation of the supplying lead-ins to the traction substations of the Moscow subway engenders up to 39 different switch-overs, necessitating the same number of tele-signals from the dispatcher's panel [1].

In these conditions, circular systems, to a large degree, meet the requirements of maximum clarity of representation, on the dispatcher's panel, of the occurrences at the controlled points, and minimum duration of signal transmission for switch-overs (not isolated ones) requiring the attention or intervention of the dispatcher.

Precisely this requirement for circular transmission of information signals was predetermined in a host of cases making use of the distributive method of selection in TC-TS devices with time-separated signals.

Ordinarily, for remote control connections in remote-controlled industrial and transport enterprises, independent channels are used (in the remote control circuits) and, therefore, the question of transmission effectiveness and rapidity of action of TC-TS devices was not sharply raised until quite recently. However, with the increase in speed of the controlled and regulated objects themselves, and with the increase in the number of objects included in the channel, this question acquired ever greater importance and forced a reconsideration of the previously accepted solutions.

With respect to speed and effectiveness of transmission of individual signals, the distributive method of selection is inferior to other methods, since it requires the expenditure of one code element for each signal which could be transmitted. In other words, with the distributive method of selection, the code must contain a number of elements (pulses or intervals) equal to the number of signals, although, as stated above, in any one cycle of tele-control, only one error may be transmitted.

TABLE 1

Distributive Method

Signal no.	Code elements 1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

TABLE 2

Distributed Group Method

Group no.	Signal no.	Group selection			Object selection		
		Code elements					
		1	2	3	1	2	3
1	1	1	0	0	1	0	0
	2	1	0	0	0	1	0
	3	1	0	0	0	0	1
2	4	0	1	0	1	0	0
	5	0	1	0	0	1	0
	6	0	1	0	0	0	1
3	7	0	0	1	1	0	0
	8	0	0	1	0	1	0
	9	0	0	1	0	0	1

TABLE 3

Combinatorial Method of Selection from One Set

Signal no.	Code no. 1	2	3	4	5	6
1	1	1	0	0	0	0
2	1	0	1	0	0	0
3	1	0	0	1	0	0
4	1	0	0	0	1	0
5	1	0	0	0	0	1
6	0	1	1	0	0	0
7	0	1	0	1	0	0
8	0	1	0	0	1	0
9	0	1	0	0	0	1
10	0	0	1	1	0	0
11	0	0	1	0	1	0
12	0	0	1	0	0	1
13	0	0	0	1	1	0
14	0	0	0	1	0	1
15	0	0	0	0	1	1

The formula for the code for a distributive method of selection has the form: $S = n$, where S is the total number of signals and $\underline{n}$ is the total number of code elements.

The circular possibilities of a code may be estimated by the coefficient of circularity, $\delta = s/S$, where $\underline{s}$ is the number of signals which can be transmitted in one cycle. For the distributive method of selection, $\delta = 1$.

The distributive method, in its pure form, is rarely used (for example, in the TC-TS device type VRT-53). In order to increase transmission effectiveness, the distributed group method is ordinarily employed where, for example, the first selecting pulse of the code chooses the group of objects and the second code pulse chooses the object in this group. Thanks to this, the code may contain fewer elements than there are signals. The formula for such codes has the form: $S = n_1 n_2$, where n_1 is the number of elements for group selection, equal to the number of groups, and n_2 is the number of elements for object selection in each group.

For such a code, the circularity coefficient is

$$\delta = \frac{n - n_1}{S} = \frac{n_2}{n_1 n_2} = \frac{1}{n_1}.$$

It is well known [2] that S_{max} is obtained with the condition, $n_1 = n_2 = \sqrt{S}$.

The use of the distributed group method of selection is sometimes brought about by the presence of some executive points which,in this case, are considered as independent groups of objects. The first degree of selection is then the choice of such isolated groups.

It is necessary to bear in mind that the introduction of each subsidiary group and degree of choice lowers the circular possibilities since, in circular transmission, only signals from one group may take part. Thus, for example, with two groups, circular transmission can encompass only half of all objects. In this case, the coefficient of circularity δ will equal one half.

For the analysis of methods of selection, it is convenient to represent the signals in the form of binary numbers, and the set of signals in the form of tables. Here, the selecing code element will correspond to 1, and the motion element to 0.

In Tables 1 and 2 are given the sets of signals for the distributive and the distributed group method of selection, constructed by decreasing binary numbers for $n = 6$.

TABLE 3a

Circular Combinatorial Method of Selection from One Set ("Stepping-Switch" Method of Selection)

Group no.	Signal no.	Code no. 1	2	3	4	5	6
1	1	1	1	0	0	0	0
	2	1	0	1	0	0	0
	3	1	0	0	1	0	0
	4	1	0	0	0	1	0
	5	1	0	0	0	0	1
2	6	0	1	1	0	0	0
	7	0	1	0	1	0	0
	8	0	1	0	0	1	0
	9	0	1	0	0	0	1
3	10	0	0	1	1	0	0
	11	0	0	1	0	1	0
	12	0	0	1	0	0	1
4	13	0	0	0	1	1	0
	14	0	0	0	1	0	1
5	15	0	0	0	0	1	1

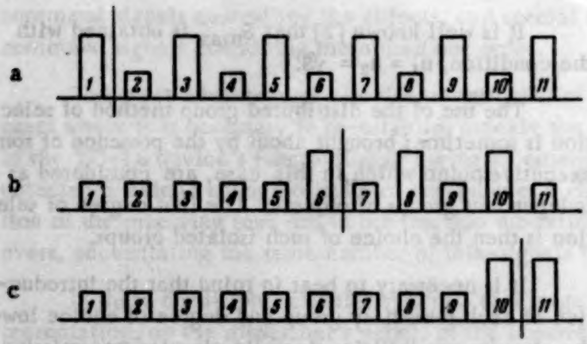

Fig. 1. Circular code for the "stepping-switch" method of selection: a) choice of the second, fourth, sixth and eighth objects from the first group, b) choice of second and fourth objects of the sixth group, c) the selection of the one object in the tenth group.

In Table 3 is given the set of signals for the combinatorial method of selection from one set with a choice of two, also for n = 6. The formula for a code when this method is used has the form: $S = C_n^m = C_6^2$, where m is the number of selecting elements of the code.

Here, each object is selected by means of two code elements, exactly as in the distributed group method, but there is no longer a constant division of the code elements according to the level of selection. However, in analogy to the distributed group method, it may be considered that the first selecting element selects the group of the object, and the second selects the object from its group. Using this idea, one may divide the selecting code elements into groups, as shown in Table 3a.

Considering all the combinations of Tables 3 and 3a from this point of view, one may easily convince oneself that, within the second level of choice, i. e., the level of object selection, it is completely possible to carry out circular transmission of signals without violating the principles of combinatorial selection. The boundaries of the selection groups are shown by the horizontal lines on Table 3a.

Due to the variable capacity of the groups, the circularity coefficient is different for the various groups, and varies from $\delta = (n-1)/S$ to $\delta = 1/S$ for noncircular transmission, characterizing the last group, which contains only one object.

This is also clarified by Fig. 1, on which, as examples, are given the codes for two-level selection by the "steppoing-switch" method, where the selecting marks are agreed to be taken by increasing pulse amplitude.

It should be mentioned that, for the same number of code elements, the distributed group method cannot guarantee as high a circularity coefficient as the most capacious group using the combinatorial method since, even with just two groups, the circularity coefficient δ, for the distributed group method, equals $(n-2)/S$.

Using the combinatorial method of selection from one set with a three-fold choice (analogous to a three-level application of the distributed group method), one may consider that the first selecting code element selects the group of the object, (for example, the group of isolated objects, i. e., the controlling points), the second element selects the subgroup of this group, and the third element selects the object in the subgroup.

Extending this to arbitrary combination of n things taken m at a time, one may write the general expressions for the number of signals, the number of groups and the number of objects in the groups, and also the circularity coefficient for a given number of code elements for such a circular combinatorial method of selection from one set which, due to the way in which it was developed, was called the "stepping-switch" selection method.*

* The "stepping-switch" selection method was laid down, and developed, by K. P. Kordiukov and L. I. Kazanskii.

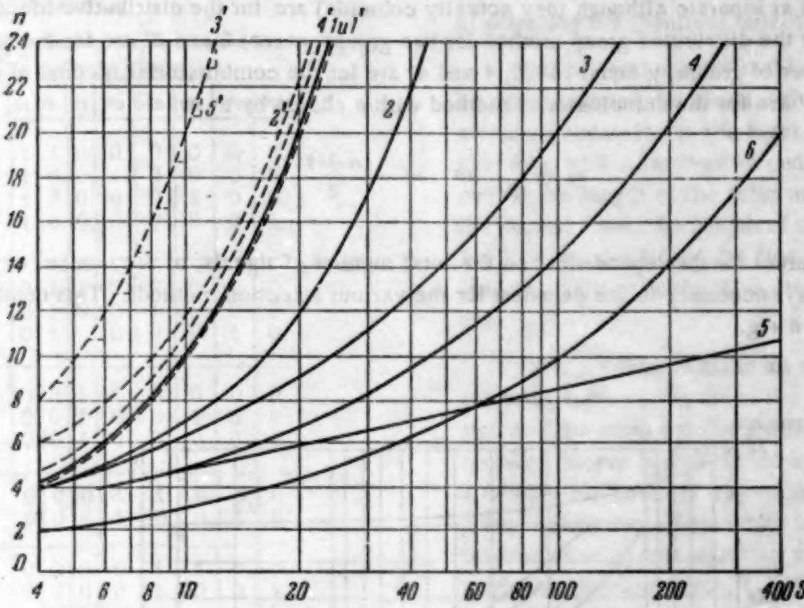

Fig. 2. Dependence of the number of code pulses on the number of signals, for different methods of selection.

The total number of signals (the code formula) is $S = C_n^m$. The number of groups is $g = C_{n-1}^{m-1}$. The number of signals, s, in the group varies from $n - m + 1$ to 1. The circularity coefficient, δ, varies from

$$\frac{n-m+1}{S} \text{ to } \frac{1}{S}.$$

Figure 2 gives curves characterizing the selection methods considered here. The total number of signals is laid out along the abscissa and along the ordinate axis, the number of pulses which must be contained in the code in order to transmit these signals. This quantity can also characterize the duration of transmission, if the duration of the code elements is known and if the difference in duration between the selection and motion elements is ignored. The dotted lines characterize the number of signals which may be transmitted circularly in

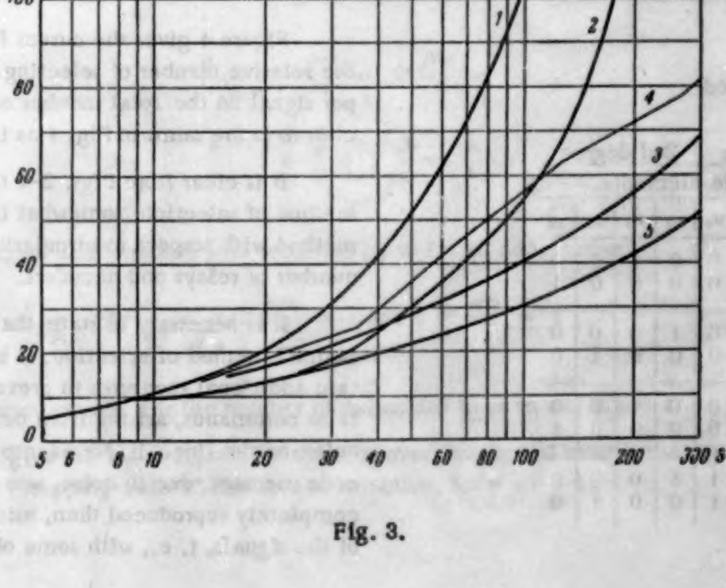

Fig. 3.

one cycle where, for the "stepping-switch" method, the number of signals of the first group is stated. On Fig. 2, curves 1 and 1' (shown as separate although they actually coincide) are for the distributive method of selection, curves 2 and 2' are for the distributed group method for two groups, curves 3 and 3' are for the distributed group method with the number of groups, g equal to $\sqrt{S}$, 4 and 4' are for the combinatorial method of choice by two elements, and 5 and 5' are for the combinatorial method with a choice by m, where

$$m = \frac{n}{2} \quad \text{or} \quad m = \frac{n \pm 1}{2}.$$

Figure 3 gives curves for the dependence, on the total number of signals, of the number of counting, grouping and composing relays necessary in the decoders for the various selection methods. This number is calculated from the formula, K = n + g.

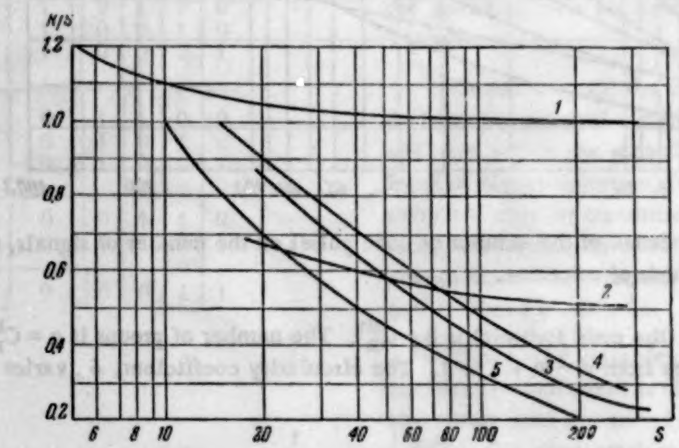

Fig. 4.

Here, curve 1 is for the distributive method, curve 2 for the distributed group method for two groups, curve 3 for the distributed group method with the number of groups equal to $g = \sqrt{S}$, curve 4 is for the "stepping-switch" method with a selection by m and curve 5 for the "stepping-switch" method with a choice by two.

The number of executive relays for this was not counted, since the number of them does not depend on the method of selection, and is always proportional to the number of objects. Likewise, the number of protective relays, the number of which depends on the structure and design of the device, was not calculated.

TABLE 4

Distributed Group Method

Group no.	Signal no.	1st degree				2nd degree			
		Code elements							
		p_1	n_1	p_2	n_2	p_3	n_3	p_4	n_4
1	1	1	0	0	0	0	1	0	0
	2	1	0	0	0	0	0	0	1
2	3	0	1	0	0	1	0	0	0
	4	0	1	0	0	0	0	1	0
3	5	0	0	1	0	0	1	0	0
	6	0	0	1	0	0	0	0	1
4	7	0	0	0	1	1	0	0	0
	8	0	0	0	1	0	0	1	0

Figure 4 gives the curves for the dependence of the relative number of selecting and distributing relays per signal on the total number of signals. The nomenclature is the same in Fig. 4 as in Fig. 3.

It is clear from Figs. 2-4 that the "stepping-switch" method of selection, somewhat inferior to the distributive method with respect to circularity, requires the lowest number of relays and decoders.

It is necessary to state that, in using the "stepping-switch" method of selection, it is necessary to take certain additional measures to prevent the execution of false commands, arising from defective devices or from noise on the line. If, for example, the first selecting code element, due to noise, was lost or somehow incompletely reproduced then, using circular transmission of the signals, i. e., with some object-selecting element

TABLE 5

"Stepping-Switch" Method

Group no.	Signal no.	p_1	n_1	p_2	n_2	p_3	n_3	p_4	n_4
1	1	1	1	0	0	0	0	0	0
	2	1	0	0	1	0	0	0	0
	3	1	0	0	0	0	1	0	0
	4	1	0	0	0	0	0	0	1
2	5	0	1	1	0	0	0	0	0
	6	0	1	0	0	1	0	0	0
	7	0	1	0	0	0	0	1	0
3	8	0	0	1	1	0	0	0	0
	9	0	0	1	0	0	1	0	0
	10	0	0	1	0	0	0	0	1
4	11	0	0	0	1	1	0	0	0
	12	0	0	0	1	0	0	1	0
5	13	0	0	0	0	1	1	0	0
	14	0	0	0	0	1	0	0	1
6	15	0	0	0	0	0	1	1	0
7	16	0	0	0	0	0	0	1	1

in the code, the first element would seem capable of selecting an incorrect group, and the second element of selecting the incorrect object from this group.

For protection against this eventuality, there might be recommended the use of different type pulses or code elements according to the level of selection. Thus, for example, with a time-pulse code, for the first level of choice the length of the pulse might be used and, for the second level, the length of the pause, or vice versa. In this case, the loss of the first selecting sign would lead to an illegal code, i. e., to the prevention of failures.

It is possible to make an even fuller use of the selecting signs when part of the group is chosen by one sign and the other part by a different sign. The corresponding object is also chosen by the two signs but in a completely contrary way to that in which the given group was chosen. This might be called the crossover method of using two selecting signs. The signs themselves might be interdependent, i. e., be such that, under the action of noise, can be converted one to the other as occurs, for example, in the time-pulse code using the length of pulse and pause. The cross over method of using them guarantees complete protection from the execution of signals thus distorted.

It was found [3] that the total number of signals (i. e., the code formula) and the number of groups for the crossover use of pulses and pauses are expressed in the following way:

for the distributed group method

$$S_{p,n} = p_1 n_2 + n_1 p_2, \quad g = p_1 + n_1;$$

for the "stepping-switch" method with selection by two

$$S_{C^2_{p,n}} = C^2_{p+1} + C^2_n \text{ and } g = C'_p + C'_{n-1};$$

for the "stepping-switch" method with selection by m (m an even number)

$$S_{C^m_{p,n}} = C^m_{p+\frac{m}{2}} + C^m_{n+\frac{m}{2}-1},$$

$$g = C^{m-1}_{p+\frac{m}{2}-1} + C^m_{n+\frac{m}{2}-2};$$

for the "stepping-switch" method with selection by m (m an odd number)

$$S_{C^m_{p,n}} = C^m_{p+\frac{m-1}{2}} + C^m_{n+\frac{m-1}{2}} \text{ and } g = C^{m-1}_{p+\frac{m-1}{2}-1} + C^{m-1}_{n+\frac{m-1}{2}-1},$$

where p is the number of pauses, n is the number of pulses and m is an odd number.

Tables 4 and 5 give the set of signals for the crossover method of using pulse signs for, respectively, the distributed group and "stepping-switch" methods of selection, for p = n = 4.

The curve, S = $f(p, n)$, for the crossover method of using pulse signs with a "stepping-switch" method of selection is shown in Fig. 2 (curve 6). It should be mentioned that, with the use of the crossover method and the "stepping-switch" selection method, the number of signals grows to more than twice (as occurs, for example, for the distributed group method) in comparison to the separate use of one or another sign.

SUMMARY

The "stepping-switch" method of selection may be considered as a variation of the distributed group method in which there is no constant division of the code elements by level of selection (apart from the $m - 1$ first and successive elements), or as a variation of the combinatorial method of selection from one set in which is guaranteed circular transmission of signals in the group selected by the first $m - 1$ selecting code elements.

The "stepping-switch" method of selection combines in itself the positive qualities of the distributive method (circularity) and the combinatorial method from one set (higher transmission effectiveness).

The "stepping-switch" method of selection by two requires the least outlay of relays and decoders.

The "stepping-switch" method of selection together with the crossover method allows the use of two interdependent pulse signs, which, with full protection of the signals, more than doubles the effectiveness of transmission.

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CONCERNING ONE TYPE OF COMMUTATION CIRCUIT

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One type of commutation circuit is investigated — a circuit for converting binary numbers for every possible permutation of their terms. The method of synthesis, based on the use of the theory of permutation groups, provides the possibility of obtaining optimal circuits for a small expenditure of trouble on their components. The formula will be provided for the computation of the number of contacts necessary for the construction of the commutation circuit considered.

Together with the general methods of synthesizing relay-contact circuits [1-4] there exist particular methods for individual types of circuits, allowing significant simplifications and increases in speed of the process of planning these types of circuits. The present notes on the construction of a commutator present one such method, which makes use of certain results from the theory of groups.*

The problem of constructing a commutator arose in connection with the development of a machine for the synthesis of contact circuits [6], modeling the method of cascades [3] (the graphical method of [4]).

It is well known that the number of contacts in a circuit which has been synthesized by the method of cascades, and certain other of its parameters, depend on the order of synthesis [3], i. e., on the order in which the selected elements follow one another. If the number of such elements is $\underline{n}$ then, to obtain the optimal (for the given method of synthesis) circuit, it is necessary, in the most general case, to examine all $n!$ circuits obtained as a result of carrying out all possible permutations of the numbering of the variables in the decomposition into constituent units of the function to be realized.

The equivalence of the problem of permuting the numbering of the variables with the problem of substituting binary numbers becomes obvious, amounting only to the decomposition of the function to its constituent units whereby each nonnegated variable is treated as a 1, and each negated variable as a 0.

We remark that an arbitrary permutation of the terms of an n-place binary number does not change the number of ones and zeroes in the number. Therefore, all 2^n n-place binary numbers may be divided into n + 1 classes and, consequently, all commutators may be divided into n + 1 sections (Fig. 1), adopting the number $\underline{r}$ for that sector assigned to binary numbers containing $\underline{r}$ ones. The class numbered $\underline{r}$ contains C_n^r binary numbers. For an arbitrary nonidentical permutation of terms, the binary numbers, in some fashion, change places within the classes, since there is no pair of terms with the same elements in all the numbers.

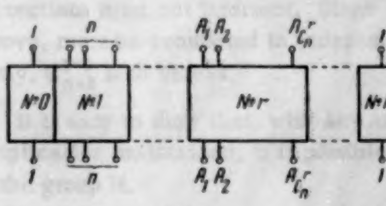

Fig. 1.

Thus, to each permutation of order $\underline{n}$ of the terms there corresponds one and only one permutation, of order C_n^r, of the binary numbers themselves.

* A particular case of the circuit presented was studied in [5].

The following assertions hold.

1) The permutations of the binary numbers, obtained by the permutation of their terms, form a group isomorphic to the symmetric group $\mathfrak{S}_n$ [7].

In fact, let the binary number A_i, under the identity permutation of terms, with the ordinal numbering 1, 2, . . ., n, have ones in the binary places:

$$\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{ir}, \quad i = 1, 2, \ldots, C_n^r, \quad 1 \leqslant \alpha_{ij} \leqslant n. \qquad (*)$$

The first subscript shows that the given "one" is to be found in binary number A_i, while the second subscript gives the ordinal numbering only for those binary places in the given number which contain "ones." The collection (*) is the set of all possible combinations of the $\underline{n}$ digits, 1, 2, . . ., n, taken $\underline{r}$ at a time and, therefore, any symbol α_{ij} is repeated in exactly C_{n-1}^{r-1} sets, i. e., the subscript $\underline{i}$ assumes C_{n-1}^{r-1} values simultaneously.

We consider the permutation of terms

$$s_i = \begin{pmatrix} 1 & 2 & \ldots & n \\ \alpha_1 & \alpha_2 & \ldots & \alpha_n \end{pmatrix}$$

(the first subscript has been omitted). It says that the "one" which was at position $\underline{p}$ now goes over to position α_p, i. e., that the "one" which was found in one of the C_{n-1}^{r-1} binary numbers is now, generally speaking, to be found in another of the C_{n-1}^{r-1} binary numbers. In this case, by repeating each pair, $\begin{pmatrix} p \\ \alpha_p \end{pmatrix}$ C_{n-1}^{r-1} times, and rearranging them in the corresponding way (so that the permutation itself is not changed), we can give the permutation s_i in the form:

$$s_i = \begin{pmatrix} \alpha_{11}\alpha_{12}\ldots\alpha_{1r}\alpha_{21}\alpha_{22}\ldots\alpha_{2r}\ldots\alpha_{C_n^r 1}\alpha_{C_n^r 2}\ldots\alpha_{C_n^r r} \\ \alpha_{i_1 1}\alpha_{i_1 2}\ldots\alpha_{i_1 r}\alpha_{i_2 1}\alpha_{i_2 2}\ldots\alpha_{i_2 r}\ldots\alpha_{i_{C_n^r} 1}\alpha_{i_{C_n^r} 2}\ldots\alpha_{i_{C_n^r} r} \end{pmatrix}.$$

Permutation s_i implies the permutation

$$S_i = \begin{pmatrix} A_1 A_2 \ldots A_{C_n^r} \\ A_{i_1} A_{i_2} \ldots A_{i_{C_n^r}} \end{pmatrix}$$

In exactly the same fashion, the permutation

$$s_j = \begin{pmatrix} \alpha_1\alpha_2 \ldots \alpha_n \\ \beta_1\beta_2 \ldots \beta_n \end{pmatrix},$$

$$s_j = \begin{pmatrix} \alpha_{i_1 1}\alpha_{i_1 2}\ldots\alpha_{i_1 r}\alpha_{i_2 1}\alpha_{i_2 2}\ldots\alpha_{i_2 r}\ldots\alpha_{i_{C_n^r} 1}\alpha_{i_{C_n^r} 2}\ldots\alpha_{i_{C_n^r} r} \\ \alpha_{j_1 1}\alpha_{j_1 2}\ldots\alpha_{j_1 r}\alpha_{j_2 1}\alpha_{j_2 2}\ldots\alpha_{j_2 r}\ldots\alpha_{j_{C_n^r} 1}\alpha_{j_{C_n^r} 2}\ldots\alpha_{j_{C_n^r} r} \end{pmatrix}.$$

implies the permutation of the binary numbers

$$S_j = \begin{pmatrix} A_{i_1} A_{i_2} \ldots A_{i_{C_n^r}} \\ A_{j_1} A_{j_2} \ldots A_{j_{C_n^r}} \end{pmatrix}.$$

Here, the product of the permutations S_i and S_j will be the permutation

$$S = S_i S_j = \begin{pmatrix} A_1 A_2 \dots A_{C_n^r} \\ A_{j_1} A_{j_2} \dots A_{j_{C_n^r}} \end{pmatrix},$$

since precisely this is implied by the permutation

$$s = s_i s_j = \begin{pmatrix} \alpha_{11}\alpha_{12} \dots \alpha\, \alpha_{12}\alpha_{22} \dots \alpha_{2r} \dots \alpha_{C_n^r 1}\alpha_{C_n^r 2} \dots \alpha_{C_n^r r} \\ \alpha_{j_1 1}\alpha_{j_1 2} \dots \alpha_{j_1 r}\alpha_{j_2 1}\alpha_{j_2 2} \dots \alpha_{j_2 r} \dots \alpha_{j_{C_n^r} 1}\alpha_{j_{C_n^r} 2} \dots \alpha_{j_{C_n^r} r} \end{pmatrix}.$$

Thus, between the permutations s of the terms and the permutations S of the binary numbers is established a one-to-one correspondence and, if permutation s_i corresponds to permutation S_i and permutation s_j corresponds to permutation S_j, then the permutation $s_i s_j$ corresponds to the permutation $S_i S_j$, which is required for there to be an isomorphism.

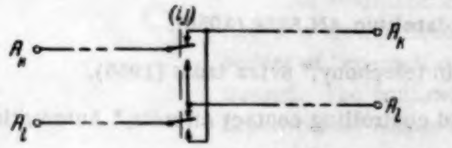

Fig. 2.

Thanks to the existence of this isomorphism, the set of all actual permutations of order C_n^r of the binary numbers constitutes a group H and $H \subset \mathfrak{S}_{C_n^r}$.

2) Every transposition $(ij) \subset \mathfrak{S}_n$ corresponds to a permutation $S_{ij} \subset H$, which decomposes into a product of independent binary cycles. This assertion follows immediately from the fact that the corresponding elements of the isomorphic groups are always of one order.

It is easy to compute the number of independent binary cycles of the permutation S_{ij}.

Indeed, under the transposition (ij), the terms are changed only in those binary numbers which contain either 0 and 1 or 1 and 0 in the transposed terms. The one and the other will be possible only the number of times that $r-1$ "ones" can be placed in the remaining $n-2$ binary places, i. e., C_{n-2}^{r-1}. This is the desired number. The converse is also obvious: all products of C_{n-2}^{r-1} independent binary cycles of the group H correspond to a transposition of the group $\mathfrak{S}_n$.

3) It is well known that the set of C_n^2 transpositions is a system forming the symmetric group $\mathfrak{S}_n$ [7]. We conclude from what was stated above that the corresponding products of C_{n-2}^{r-1} independent cycles is also a system forming a group H.

After having made these remarks, we turn to the construction of the commutator or, more accurately, to that portion of it which permutes a n-place binary number containing r "ones." We shall call this the r-th section of the commutator. The r-th section of the commutator takes the form of an orientated (C_n^r, C_n^r)-pole (Fig. 1), the input and output of which represent the binary number A_i ($i = 1, 2, \dots, C_n^r$). Conduction between input A_k and output A_l must occur each time that the permutation modeled by the commutator transforms the number A_k into the number A_l, the permutation itself being of the terms of the numbers.

By a "cross section" of the commutator we shall mean those lines passing through all contacts of one and the same relay modeling transposition (ij). Due to the noncommutativity of multiplication of permutations, the cross sections must not intersect. Since a relay is a two-position device, the contacts of this relay, by virtue of 2) above, must be connected in independent blocks, as shown in Fig. 2; for each cross section there are, consequently, C_{n-2}^{r-1} such blocks.

It is easy to show that, with any arbitrary ordering of the C_n^2 transpositions, with their permissible order of multiplication maintained, it is possible to obtain the entire symmetric group* $\mathfrak{S}_n$, and, by virtue of 3) above, also the group H.

* We mention that, in [5], it was only proven that all C_n^2 transpositions are sufficient for obtaining group $\mathfrak{S}_n$, but it follows from the method of proof that all C_{n-1}^2 transpositions which do not vary symbol n precede all the transpositions which do vary that symbol.

This means that the cross section of the r-th commutator section may be laid out in any order. Using the results of [5], one can reduce the size of the cross section of the r-th section (and, consequently, of the entire commutator).

It is easy to see that, for each binary number, there is a double for it, in the sense that wherever there is a "one" in one number there is a "zero" in the other. Therefore, the r-th and the (n − r)-th section of the commutator are implemented identically, and are isolated from each other.

If n is even, the (n/2)-th section cannot be decomposed into two isolated parts, since the double of the binary number with the same quantity of "zeroes" and "ones" may be interchanged with the number itself.

The number of switching contacts of the entire commutator is given by the formula

$$N = 2m \sum_{r=1}^{n-1} C_{n-2}^{r-1},$$

where m is the number of cross sections.

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*See English translation.

THE APPLICATION OF A THERMOELECTRIC PROPORTIONAL-PLUS-INTEGRAL CORRECTIVE DEVICE FOR IMPROVEMENT OF TWO-POSITION TEMPERATURE CONTROL

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An automatic control method which is based on application of a thermoelectric corrective device and combines the properties of two-position and proportional-plus-integral control is treated. The construction of the corrective device is described. Experimental results show that the proposed device permits a substantial improvement of the two-position control process.

The feedback-type corrective devices applicable in relay systems of automatic control are described in [1-13]. As has been shown in these works, this kind of corrective device can substantially improve the control quality of the relay systems with constant speed servomotors, as well as of the two-position control systems. The feedback systems in the form of thermoelectric corrective devices and their application with two-position temperature controllers were described in particular in [4-6]. It has been shown that this kind of device allows a sharp decrease in oscillation amplitude of the controlled variable, but at the same time they make the system static with a residual unbalance.

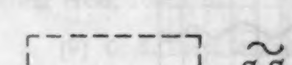

Fig. 1.

As an addition to the articles mentioned, there follows below a description of a device* which makes it practically possible to eliminate the residual unbalance and gives a system of the proportional-plus-integral type.

In the diagram of Fig. 1 the following nomenclature is used: 1 is the object of control (an installation with an electric heater), 2 is a two-position-type control unit, 3 is the corrective device, H is the heater (of the object), T is a thermocouple, K is an electric contact.

Conductor C is the basic element of the corrective device; it is made of a material different from that of the conductors connecting thermocouple T with unit 2. The ends of the conductors T_1 and T_2 are heated by heaters H_1 and H_2 which are turned on and off with heater H. Heating combination H_1-T_1 has a faster response, while H_2-T_2 has a slower response. When contact K is closed thermal emf's, which add algebraically to the thermal emf of thermocouple T, are developed at the ends of the conductors T_1 and T_2.

Junction T_1 serves to create a pulsed mode of operation, while junction T_2 serves to eliminate the residual unbalance. Due to slower response of junction T_2 it undergoes smaller temperature changes than does junction T_1. But since the heaters of both junctions are turned on and off for identical intervals of time,

* Authorship Certificate No. 104760 issued to A. A. Kampe-Nemm and A. N. Trusov.

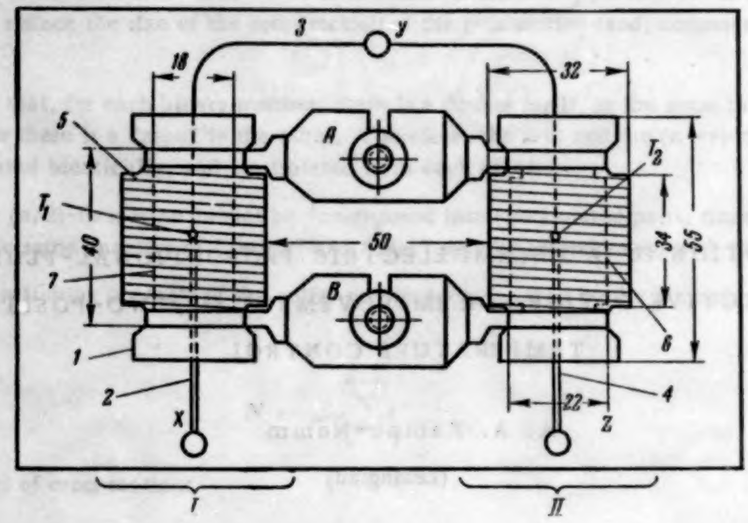

Fig. 2.

the average temperatures of the junction become the same after the transients die out; therefore, the average values of emf developed by them are cancelled out by each other and the residual unbalance is eliminated.

For the corrective device to operate properly, the extreme values of temperature (emf) to which both junctions could be heated if the heaters were energized for a longer period of time, should be the same. It is also essential that the time constants of both heating devices be sufficiently different from each other.

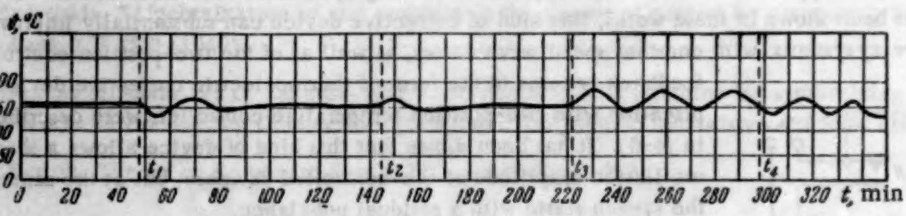

Fig. 3.

The described temperature control circuit has been experimentally tried out on an electric muffle furnace. The construction diagram of the corrective device used is shown in Fig. 2. Element I, with a faster response, consisted of two mica plates 1 and with a junction of two conductors, copper and constantan, with T_1 squeezed between them. Nichrome heater 5 was placed outside. Element II, with a slower response, differed from the faster element by the fact that an aluminum plate 6, shown by a broken line, was put between the mica plates. It was in contact with junction T_2 of two conductors, constantan 3 and copper 4. By means of binding posts A and B a voltage was applied to the heaters, while binding posts X, Y and Z served to connect the device with the thermometric circuit. In the faster element 1 a sheet of foil 7, shown by a broken line, has been placed between mica plates. It served to secure a more uniform heat transfer from the heater to the thermocouple.

Figure 3 is a graph of automatic temperature recordings obtained during the tests. Prior to instant t_3 the control, with the corrective device taking part, was in effect. The furnace doors were closed before t_1 and after t_2 and were open during the interval between t_1 and t_2. As can be seen from the graph, the controller is well able to take care of the control even under such strong disturbances. For comparison there is given, on the right of Fig. 3, a graph of the two-position control without the corrective device, where in the interval between t_3 and t_4 the furnace doors were closed and after t_4 they were open. In both instances the control process is accompanied by large oscillations.

SUMMARY

1. The application of the described thermoelectric corrective device allows a considerable decrease in the oscillation amplitude of the controlled variable, as compared with the conventional two-position control. Under undisturbed conditions the control curve becomes practically a straight line.

2. The advantage of the proportional-plus-integral corrective device over that of proportional-pulsed (static) type consists in its ability to eliminate the residual unbalance and to bring the controlled variable to the assigned value with sufficient accuracy even under strong disturbances.

3. Proportional-plus-integral thermoelectric corrective devices are simple to build and can be used as additions to two-position controllers to improve their performance.

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* Transliteration of Russian – Publisher's note.

** See English translation.

IMPROVING THE QUALITY OF TWO-POSITION CONTROL

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The operating principle and the construction of a corrective device proposed by the author for a two-position controller are considered. Some results of a graphic analysis of the process of two-position control with correction are given.

As is known, the quality of two-position control can be substantially improved by means of special corrective devices, as for example, those described in [1, 2].

In this article it is shown that the precision of a two-position controller can be increased by introducing a unique kind of interruption into the control process by means of a special corrective device of the time relay type. In the case of an extremely unsymmetrical two-position control process this kind of correction is simply realized by means of an electromechanical device, which would serve to limit the amplitude of only one (for example, positive) deviation of the controlled parameter.

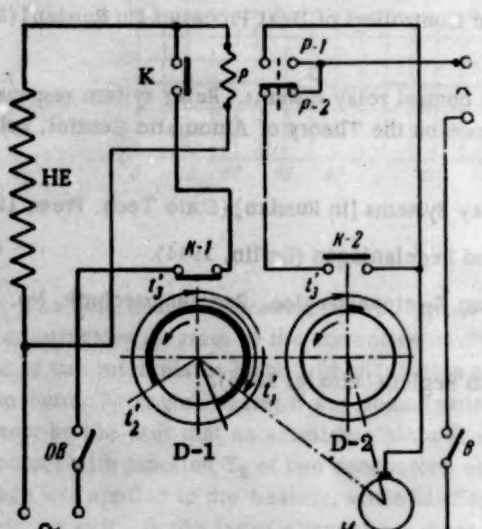

Fig. 1.

Let us note that the unsymmetrical mode of operation of two-position control takes place in many practical cases, as for example, in relatively low temperature control in thermostats, drying racks, muffle furnaces and others.

The basic diagram of an electromechanical corrective device is shown in Fig. 1.

The corrective device has a supplementary relay P with two pairs of contacts: P-2 – normally closed, and P-1 – normally open. Relay P is connected in parallel with heating element HE and is energized when control contact K is closed. The contacts of relay P send the controlling pulse to the time relay, which consists of an electric motor M driving two discs: a basic disc D-1 and a supplementary one D-2. These discs control contacts K-1 and K-2, respectively. Discs D-1 and D-2 are affixed on the same axle and their cams are so constructed that adjustment of different timing intervals is possible.

Figure 2 shows the process of two-position control for the case of a controller without a nonsensitive region (curves $\Delta\vartheta$ and $\Delta\vartheta_{s.e.}$) and a process in the same system with the corrective device operating (curves $\Delta\vartheta'$ and $\Delta\vartheta'_{s.e.}$). In Fig. 2 the following nomenclature is used: $\Delta\vartheta$ is the deviation of the controlled parameter (temperature) of the object, $\Delta\vartheta_{s.e.}$ is the temperature deviation measured by a sensitive element, $\Delta\vartheta_{m(+)}$ is the amplitude of positive deviation, $\Delta\vartheta_{m(-)}$ is the amplitude of negative deviation, T_{on} is the time on, T_{off} is the

time off, T is the period, t_1 is the instant the inflow is shut off in an ordinary control. The same quantities designated by primes correspond to a system with the corrective device, Q_{in} is the heat inflow, Q_{out} is the heat outflow, Δt is the delay time, $\Delta t'$ is the "residual" delay time with the correction process taking place, t_3' is the stopping time of the discs after the first revolution.

The operation of the corrective device is based upon the independent shutting off of heat inflow to the object an assigned time interval after the controller contact is closed.

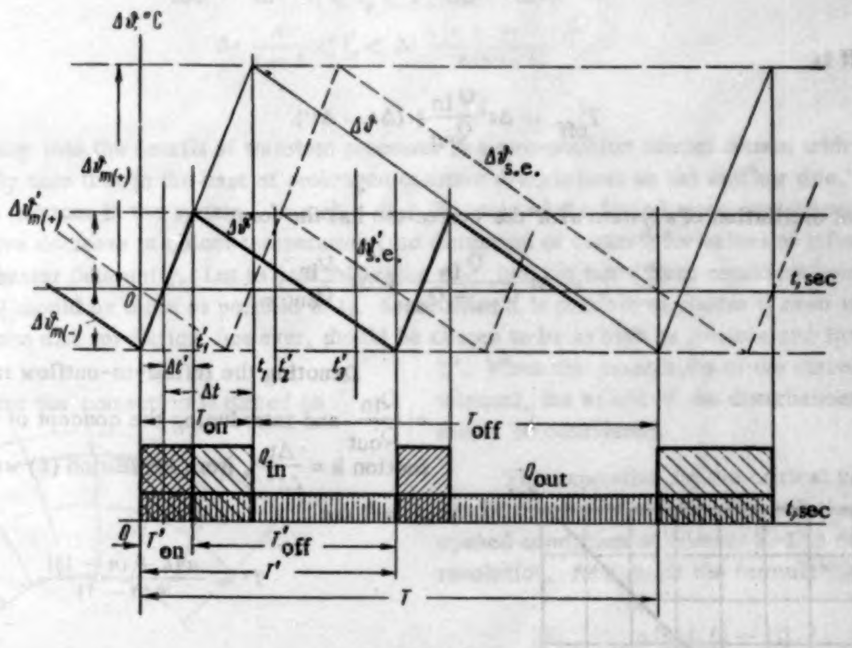

Fig. 2.

Let us consider the action of the device from the instant when the object temperature starts to fall. The temperature measured by the sensing element ($\Delta \vartheta_{s.e.}$) lags the temperature of the object. At t = 0 the controller contack K closes and energizes the supplementary relay P, which in turn energizes the electric motor of the time relay by closing its contact P-1. At the same instant the object temperature begins to rise while the temperature of the sensing element continues to fall. After an assigned time t_1' contact K-1 is opened by the cam on disc D-1 and the energy inflow to the object is stopped. At this instant the object temperature begins to fall after having passed over the assigned value and having reached its maximum $\Delta \theta_{m(+)}$. At instant t_1 the sensing element temperature reaches the assigned value and contact K closes. It is obvious that at this instant or somewhat later contact K-1 can be closed again. It is closed by cam on disc D-1 at t_2'. After t_1 the time relay continues to operate, since relay P has blocked the motor through its contact P-2 and the closed contact K-2. At some instant t_3' contact K-2 is opened by the cam on disc D-2 and the motor stops. The corrective device is ready for its next cycle.

By utilizing the simplified method of graphic analysis for computation of two-position control processes presented in [3], the expression for basic parameters of the corrected process can be obtained.

We are introducing these formulas in their final form.

The amplitude of positive deviation is

$$\Delta\vartheta'_{m(+)} = \Delta t\,\frac{Q_{in} - Q_{out}}{C}, \quad (1)$$

where C is the thermal capacity of the object.

The amplitude of negative deviation, as can be seen from the graph, remains the same:

$$\Delta\vartheta'_{m(-)} = -\Delta t\,\frac{Q_{out}}{C}. \qquad (2)$$

The time on is

$$T'_{on} = \Delta t'\,\frac{Q_{in}}{Q_{in}-Q_{out}} + (\Delta t - \Delta t')\,\frac{Q_{out}}{Q_{in}-Q_{out}}. \qquad (3)$$

The time off is

$$T'_{off} = \Delta t'\,\frac{Q_{in}}{Q_{out}} + (\Delta t - \Delta t'). \qquad (4)$$

The period of oscillation of a system with the correction has the form

$$T' = \Delta t\,\frac{Q_{in}}{Q_{in}-Q_{out}} + \Delta t'\,\frac{Q_{in}}{Q_{out}}. \qquad (5)$$

Denoting the inflow-to-outflow ratio by $n = \frac{Q_{in}}{Q_{out}}$ and introducing the concept of degree of correction $k = \frac{\Delta t}{\Delta t'}$, from Expression (5) we can obtain for $\Delta t = 1$ sec

$$T' = \frac{n\,[k + (n-1)]}{k\,(n-1)}.$$

Fig. 3.

Figure 3 shows the relation between the oscillation period T' of a system with a corrective device and the inflow-to-outflow ratio n for various values of the degree of correction k. The curves show that the oscillation frequency increases with increasing degree of correction (i. e., the precision of control), but for higher values of k the frequency increase slows down.

Parameters of a corrective device under steady state conditions can be selected by means of the formulas derived above. In doing this, it is necessary to begin with the assigned precision of control, i. e., the admissible value of $\Delta\vartheta'_{m(+)}$. Obviously the degree of correction is determined as

$$k = \frac{\Delta\vartheta_{m(+)}}{\Delta\vartheta'_{m(+)}}.$$

The time when contact K-1 is closed is obtained from Formula (3) and the expressions for n and k:

$$T'_{on} = t'_1 = \Delta t\left[\frac{n + (k-1)}{k\,(n-1)}\right].$$

The time when the contact is open is found from the formula

$$t_{open} = \Delta t\,\frac{k-1}{k}.$$

For the minimal time value of the new closing of the contact, we have

$$t_2' = t_1 = \Delta t \frac{n}{n-1}.$$

According to the curves of Fig. 2, the time of one revolution of the discs under steady state conditions must satisfy the inequality

$$t_1 < t_3' < T' \quad \text{or}$$

$$\Delta t \frac{n}{n-1} < t_3' < \Delta t \frac{n\,[k+(n-1)]}{k\,(n-1)}.$$

Without going into the details of transient processes in a two-position control system with a corrective device, we shall only note that in the case of prolonged constant disturbances on the outflow side, there can appear complex periodic motions in the system. In such a case, because of the forced temporary circuit break, there will be an excessive decrease in object temperature (the disruption of control) for values of inflow-to-outflow ratio somewhat greater than unity. Let us call this value n_{cr}. Judging from these considerations, the instant of the new closure t_2' should be close as possible to t_1. Sometimes it is possible to choose t_2' even somewhat less than t_1. The time of one disc revolution, however, should be chosen to be as high as possible and little different from T'. When the parameters of the corrective device are selected, the nature of the disturbances of the system should be considered.

The instant the correction is turned on

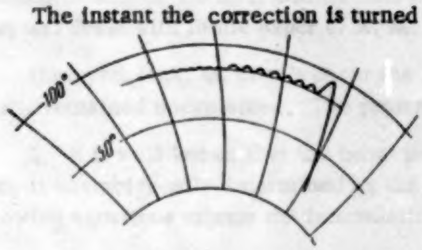

Fig. 4.

The expression for the critical value of ratio n_{cr} can be obtained from the time relation of the closed and opened conditions of contact K-1 in one complete disc revolution. As a result the formula*

$$n_{cr} = \frac{n\,[k+(n-1)]}{n^2+(k-1)}$$

is obtained where <u>n</u> is the nominal value of the inflow-to-outflow ratio (under steady state conditions).

The experimental check on the proposed method was conducted with a system where the controlled object temperature was that of a drying cabinet.

The temperature was controlled and recorded by means of an electronic potentiometer EPD-12 and a chromel-copel thermocouple without a jacket.

The process of ordinary two-position control, the process of two-position control with correction and the parameters of the corrective device were calculated.

The given design data are: the power rating of the cabinet N = 1 kw, the thermal capacity C = 4 kcal/deg, delay time Δt = 90 sec, inflow-to-outflow ratio n = 6.

The following basic parameters of the two-position control process were obtained by calculation: positive deviation amplitude $\Delta \vartheta_{m(+)} = 4.5°C$, negative amplitude $\Delta \vartheta_{m(-)} = 0.9°C$, period T = 10.8 min.

It was necessary to secure ± 1°C accuracy of the temperature regulation in the cabinet.

The basic parameter values obtained for the process with correction were: positive deviation amplitude $\Delta \vartheta'_{m(+)} = 1°C$, degree of correction k = 4.5, negative amplitude $\Delta \vartheta_{m(-)} = 0.9°C$, time on T_{on} = 0.64 min, period T' = 3.84 min.

The accepted parameters of the corrective device were as follows: time off t_1' = 0.7 min, time on t_2' = = 1.7 min, time of one revolution t_3' = 3.5 min.

The experimental record of this example is shown in Fig. 4.

* The formula is derived for maximum possible time of one revolution, i. e., for $t_3 = T_1$.

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LETTERS TO THE EDITOR

ON THE RELATIONSHIPS BETWEEN THE ERROR COEFFICIENTS AND THE AMPLITUDE AND PHASE CHARACTERISTICS OF LINEAR REPRODUCING SYSTEMS

V. G. Vasil'ev

(Moscow)

Expositions of the connections between the error coefficients and the other parameters and characteristics which describe reproducing systems have appeared in a number of works [1-3]. In particular, the question of the interrelationship of the error coefficients and the system's frequency characteristics (imaginary, real and amplitude) was dealt with in the paper of A. M. Boev.

However, in A. M. Boev's paper the interrelationship of the error coefficients and the system's phase characteristic remained unexplained. The present note establishes the character of this relationship.

1. It is well known that the behavior of the system's transfer function along the imaginary axis of the p-plane is unambiguously determined by the system's amplitude and phase characteristics, $M(\omega)$ and $\varphi(\omega)$. The following equations express the interrelationships of these characteristics with the system's transfer function, $K(p)$:

$$K(\pm j\omega) = M(\omega)\, e^{\pm j\varphi(\omega)}, \tag{1a}$$

$$M(\omega) = \sqrt{K(j\omega)\, K(-j\omega)}, \tag{1b}$$

$$\varphi(\omega) = \frac{1}{2j} \ln \frac{K(j\omega)}{K(-j\omega)}. \tag{1c}$$

For sufficiently small values of ω, the system's amplitude and frequency characteristics can be represented by series expansions:

$$M(\omega) = \sum_{k=0}^{\infty} \frac{1}{k!} M^{(k)}(0)\, \omega^{k}, \tag{2a}$$

$$\varphi(\omega) = \sum_{k=0}^{\infty} \frac{1}{k!} \varphi^{(k)}(0)\omega^{k}. \tag{2b}$$

We investigate the structure of the component of the formula, $K(j\omega) = M(\omega)e^{j\varphi(\omega)}$, in the neighborhood of the point $\omega = 0$.

It follows from the analyticity of the system's transfer function in the neighborhood of the point $p = 0$ that

$$\lim_{p\to 0} \frac{d^k}{dp^k} K(p) = \lim_{j\omega\to 0} \frac{d^k}{d(j\omega)^k} K(j\omega) = K^{(k)}(0).$$

In accordance with Formula (1a), this allows us to write the following equality:

$$\sum_{k=0}^{\infty} \frac{j^k}{k!} K^{(k)}(0)\omega^k = \sum_{k=0}^{\infty} \sigma_k \omega^k \sum_{k=0}^{\infty} M_k \omega^k, \tag{3}$$

where

$$\sigma_k = \frac{1}{k!} \lim_{\omega \to 0} \frac{d^k}{d\omega^k} [e^{j\varphi(\omega)}]$$

$$M_k = \frac{1}{k!} \lim_{\omega \to 0} \frac{d^k}{d\omega^k} M(\omega).$$

In order to express the coefficients σ_k, by means of the coefficients $\varphi_k = \frac{1}{k!} \varphi^{(k)}(0)$, we apply Leibnitz' formula to the right side of the equation:

$$\frac{1}{k!} \lim_{\omega \to 0} \frac{d^k}{d\omega^k} [e^{j\varphi\omega}] = \frac{j}{k!} \lim_{\omega \to 0} \frac{d^{k-1}}{d\omega^{k-1}} \left\{ e^{j\varphi(\omega)} \frac{d}{d\omega} [\varphi(\omega)] \right\}.$$

This allows one to write that, for $k > 0$,

$$\sigma_k = j\left(\frac{1}{k} \sigma_{k-1}\varphi_1 + \frac{2}{k} \sigma_{k-2}\varphi_2 + \ldots + \frac{k-1}{k} \sigma_1 \varphi_{k-1} + \varphi_k \right). \tag{4}$$

Turning back to Expression (3), we notice that

$$\frac{j^k}{k!} K^{(k)}(0) = \sigma_k M_0 + \sigma_{k-1} M_1 + \ldots + \sigma_1 M_{k-1}. \tag{5}$$

Analysis of the right side of this equation shows that it is the expansion of the determinant

$$G_k = \begin{vmatrix} M_0 & (-1)M_1 & (-1)^2 M_2 & \ldots & \ldots & \ldots & (-1)^k M_k \\ 1 & \left[j\frac{1}{k}\varphi_1\right] & \left[(-1)j\frac{2}{k}\varphi_1\right] & \ldots & \ldots & \ldots & [(-1)^{k-1} j\varphi_k] \\ 0 & 1 & \left[j\frac{1}{k-1}\varphi_1\right] & \ldots & \ldots & \ldots & [(-1)^{k-2} j\varphi_{k-1}] \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & \ldots & \left[\frac{1}{3} j\varphi_1\right] & \left[(-1)\frac{2}{3} j\varphi_2\right] & [(-1)^2 j\varphi_3] \\ 0 & \ldots & \ldots & \ldots & 1 & \left[\frac{1}{2} j\varphi_1\right] & [(-1) j\varphi_2] \\ 0 & \ldots & \ldots & \ldots & 0 & 1 & [j\varphi_1] \end{vmatrix}$$

by the elements of the first row.

2. Computation of the coefficients σ_k and G_k is essentially simplified if it is borne in mind that, for odd values of $\underline{k}$, $M_k = \frac{1}{k!} M^{(k)}(0) = 0$ and, for even values of $\underline{k}$, including $k = 0$, $\varphi_k = \frac{1}{k!} \varphi^{(k)}(0) = 0$.

SUMMARY

Expressions were obtained relating the error coefficients with the Cauchy coefficients for the system's amplitude and phase characteristics.

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REVIEW

ELECTRICAL DEVICES FOR SOLVING ALGEBRAIC EQUATIONS

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The existing root finder circuits are reviewed, classified and evaluated from the point of view of the possible automatization of the root finding process.

INTRODUCTION

Many approximate methods for determining numerical values of roots of higher order algebraic equations are extremely labor consuming, hence, for the purpose of speeding up and facilitating the finding of roots, computers which can be called root finders are being developed in many countries.

The most perfect of these devices find roots automatically and display the results in the form of a root distribution picture on a complex plane. The screen of a cathode-ray tube usually serves as the complex plane in such root finders; the roots appear on the screen as luminous points, whose coordinates correspond to those of the roots.

By observing the root distribution picture of a characteristic ACS (Automatic Control System) equation, one can select coefficients which would ensure stability and certain qualitative indexes, such as the degree of stability, tendency to oscillate, etc. It is also relatively simple to find by means of root finders the boundaries of stability both with respect to one coefficient and to the plane of two coefficients. It is possible to observe the migration of roots and traces their trajectory by changing the coefficients of the equation set up on the root finder.

Besides their basic purpose root finders can be used for evaluating polynomials with scalar and complex variables, for the analysis and synthesis of Fourier series, for the solution of transcendental equations whose terms are represented by power series, etc. It is easy to make up on the basis of root finders devices for conformal mapping, used in designing automatic control systems.

There exist mechanical, hydraulic, electrical, electrolytic and other root finders. Only electrical devices* are dealt with in this article, since they are at present the most perfectly developed.

I. General Conditions

In a general case roots of an algebraic equation

$$w(p) = a_n p^n + a_{n-1} p^{n-1} + \ldots + a_\nu p^\nu + \ldots + a_1 p + a_0 = 0 \qquad (1)$$

can be represented in a polar or cartesian system of coordinates in one of the three forms of notation:

the exponential form

$$p = \rho e^{j\theta}; \qquad (2)$$

* As far as the author knows one of the first electrical root finder circuits was developed by K. V. Filatov [1].

the trigonometrical form

$$p = \rho\,(\cos\theta + j\sin\theta); \tag{3}$$

the algebraic form

$$p = x + jy. \tag{4}$$

When above expressions are introduced into the original equation three forms are obtained for the polynomial $\underline{w}$:

$$w = a_n \rho^n e^{jn\theta} + \ldots + a_\nu \rho^\nu e^{j\nu\theta} + \ldots + a_1 \rho e^{j\theta} + a_0, \tag{5}$$

$$w = \sum_{\nu=0}^{n} a_\nu \rho^\nu \cos\nu\theta + j \sum_{\nu=0}^{n} a_\nu \rho^\nu \sin\nu\theta = U + jV, \tag{6}$$

$$w = a_n (x + jy)^n + \ldots + a_\nu (x + jy)^\nu + \ldots + a_1 (x + jy) + a_0 = U(x, y) + jV(x, y). \tag{7}$$

Let us call Form (5) exponential, Form (6) trigonometrical and Form (7) algebraic. The coefficients as well as the roots of the polynomial can be complex quantities.

The basic principle of operation of electric root finders consists in selecting such values for ρ and θ (or $\underline{x}$ and $\underline{y}$) which convert polynomial $\underline{w}$ into zero. In the trignometrical and algebraic forms of expression both the real part U and the imaginary part V of the polynomial $\underline{w}$ must become zero simultaneously. Root selection can be carried out either by the method of successive approximations or by inspecting the plane of the roots.

The method of successive approximations consists in changing alternately coordinates ρ and θ ($\underline{x}$ and $\underline{y}$) in such a way as to decrease the modulus of polynomial $\underline{w}$ with each change. The coordinates corresponding to the zero value of $\underline{w}$ are those of the root.

The evaluation of the root by successive approximations can be carried out either manually or automatically. In the manual method the root finder operator, while observing quantity $|w|$, changes the coordinates by hand striving to reduce $|w|$ to zero. In the automatic method the operator does not take part in the evaluation process and the required changes of the coordinates are obtained by means of feedback between $\underline{w}$ and the coordinates ρ and θ ($\underline{x}$ and $\underline{y}$). In this case the root finder becomes a closed automatically controlled system.

The method of root plane inspection consists in moving variable $\underline{p}$ over the complex root plane along a fixed trajectory covering all the points of the area under investigation, and in clamping those values of $\underline{p}$ which convert $\underline{w}$ into zero.

The evaluation of roots by this method can also be carried out manually or automatically. In the manual method variable $\underline{p}$ usually describes on the root plane (p-plane) a circle, with its center at the origin of coordinates, the radius of which the operator has to change by hand. To this circular movement of point $\underline{p}$ corresponds the movement of point w(p) over plane $\underline{w}$ along a closed trajectory known as the polar curve. When $\underline{p}$ passes through a point which corresponds to a root of a given polynomial, the trajectory w(p) crosses the origin of coordinates on plane $\underline{w}$. In the manual method the operator must, therefore, observe the polar curve and change radius ρ so as to make the curve pass through the origin of coordinates. The corresponding value of the radius is modulus ρ of the required root. Root finders in which the evaluation of the roots is done manually by the inspection method can be called semi-automatic.*

In the automatic method the point determining variable $\underline{p}$ is given a continuous spiral motion around the origin of coordinates in plane $\underline{p}$ without any assistance from the operator. Usually a part of the spiral trajectory limited by a minimum and maximum radii is chosen so that the investigated area of plane $\underline{p}$ has the shape of a ring. Although the spiral trajectory does not pass through all the points of the ring area, it is possible to increase

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* In foreign literature such root finders are often called isographs.

the density of spiral winding to such an extent that, with some of the spaces remaining insensitive to the zero indicator, all the roots lying in that area will be clamped.

In electrical root finders of any type, owing to their physical limitations, unity is usually taken as the upper limit for the modulus of variable p. The lower limit is determined by accuracy requirements and is often taken as 0.1. For determining roots lying outside above limits the equation is transformed by changing variable p. The following substitutions are used:

$$p = kS, \quad p = \frac{k}{S}, \tag{8}$$

where k is the constant coefficient and S the new variable.

In order not to complicate the root finder circuit, the coefficients of the equation to be solved must not exceed unity, which is easily attainable if the original equation is divided by its largest coefficient.

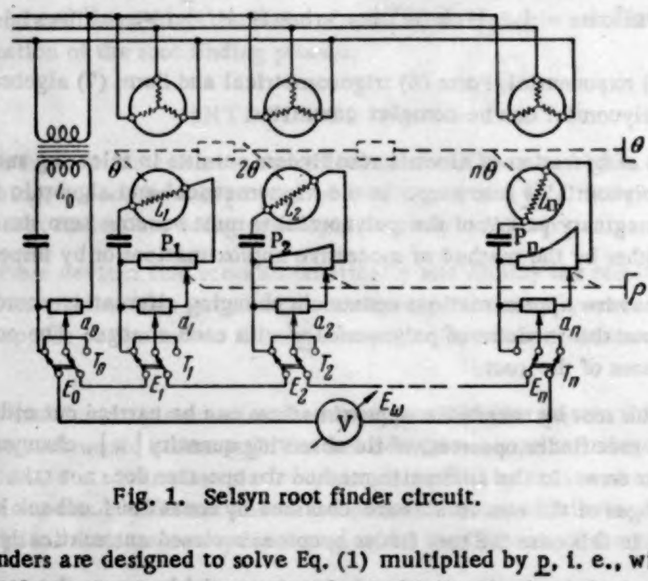

Fig. 1. Selsyn root finder circuit.

Some of the root finders are designed to solve Eq. (1) multiplied by p, i. e., without a free term. This is done with the object of unifying circuit components. The multiplication by p does not change, of course, the roots of the equation, but simply adds one zero root which can be subsequently omitted. In describing concrete root finders below it will not be stated whether they solve equations with or without a free term.

An analytical examination shows that the method of inspecting a complex root plane has a number of advantages over the successive approximations method. Thus in manually operated root finders the evaluation of roots by the inspection method is much faster than by the approximation method. The actual working of an automatic root finder based on the inspection method at the Institute of Automation and Remote Control of the Academy of Science, USSR has shown that it can deal with a wider range of problems including the finding of roots of algebraic equations.

In electrical root finders the terms of polynomial w are usually represented by tensions. The characteristics of the calculating part of the instrument which produces these tensions are determined by the form in which the polynomial w is presented, whether it is (5), (6) or (7). The basic classification of root finders, therefore, will depend not only on the methods and means of finding roots, but also on the polynomial form which the calculating part of the instrument handles. By the latter characteristic the existing devices can be divided into three types: 1) exponential form polynomial root finders, 2) trigonometrical form polynomial root finders and 3) algebraic form polynomial root finders.*

* Algebraic form polynomial root finders are usually employed as analog computers adapted for solving equations [23, 24]. Hence, they have been excluded from further consideration.

II. Exponential Form Polynomial Root Finders

In the early electrical root finders, based on the exponential form polynomial $\underline{w}$, the evaluation of roots was carried out manually by the successive approximation method. Similar root finder circuits were proposed by Hart [2], Schooley [3], Elizarenkov [4] and Ginsburg [5]. As an example let us examine Elizarenkov's circuit (Fig. 1).

The selsyn rotors $L_1, L_2, L_3, \ldots, L_n$ are interconnected by means of a gear assembly in such a manner that when rotor L_1 is turned over an angle θ, rotor L_2 turns over 2θ, etc. Rotor L_n turns over an angle of $n\theta$. The selsyn stators are fed from a three-phase supply of frequency f. The rotor windings are connected to the modulus setting potentiometers $P_1, P_2, \ldots, P_n$.

The relation between the output tension and position of the slider on potentiometer P_1 is linear, on P_2 quadratic, on P_n of the $\underline{n}$th power. The potentiometer slides are connected to a common axis turned by means of handle ρ over equal angles. The slides are connected to linear coefficient setting potentiometers $a_1, a_2, \ldots, a_n$. The circuit also contains a transformer whose secondary winding L_0 is connected to the a_0 coefficient setting potentiometer. The circuit is pre-set in such a way that, in the zero position of handle θ, all tensions in the selsyn rotors and transformer winding L_0 are in phase and equal in amplitude.

Tensions at the output of the potentiometers $a_0, a_1, \ldots, a_n$, in any arbitrary position of handles θ and ρ, can be expressed by means of the symbolic method, well known in electrical engineering as:

$$E_0 = Aa_0e^{j2\pi ft},\ E_1 = Aa_1\rho e^{j(2\pi ft+\theta)}, \ldots, E_n = Aa_n\rho^n e^{j(2\pi ft+n\theta)}. \tag{9}$$

The sum of these tensions E_w can be expressed as

$$E_w = A\,(a_n\rho^n e^{jn\theta} + \ldots + a_1\rho e^{j\theta} + a_0)\,e^{j2\pi ft}. \tag{10}$$

Tension E_w equals zero when $a_n\rho^n e^{jn\theta} + \ldots + a_1\rho e^{j\theta} + a_0 = 0$, i. e., when ρ and θ assume values of Eq. (1) root coordinates. The summation of tensions $E_0, E_1, \ldots, E_n$ is carried out by connecting the coefficient potentiometers in series. The zero value of the sum is observed by the operator on voltmeter V. Tumbler switches T_0 to T_n serve to place signs before coefficients a_0 to a_n.

The differences between this circuit and those mentioned in [2, 3 and 5] are unessential. Instead of selsyns Hart and Schooley used ac generators. In Ginsburg's instrument the polynomial $\underline{w}$ is set up in the calculating part of the device in its exponential form, but is later divided into its real and imaginary components, which provides an opportunity of representing $\underline{w}$ as a point on a cathode-ray tube screen, simulating plane $\underline{w}$.

The accuracy of root evaluation in the selsyn circuit is 2-10% for the modulus and 3-5° for the angle. The sources of error include, in addition to the variation in circuit components, distortion of the carrier waveform and unbalance between the selsyn phase supply voltages.

In 1955 Parker and Williams [6] constructed on the basis of the same circuit a more accurate instrument, using high quality components (magslips, spiral potentiometers, summation amplifiers) and a special circuit for eliminating supply phase unbalance and carrier distortion, which reduced the error to fractions of one percent. The gear assembly was considerably simplified by making some of the selsyns rotate in the opposite direction to the others. The powers of ρ were obtained by a cascade connection of linear potentiometers, separated by amplifiers.

Bubb [7] developed a root finder circuit essentially different from the above. His circuit is based on linear potentiometers and operational amplifiers, i. e., high gain amplifiers with a large negative feedback. As is known the input voltage E_i and output voltage E_0 of such amplifiers bear the relation

$$E_i = -\frac{z_2}{z_1}E_0\ , \tag{11}$$

where z_1 is the input impedance and z_2 the feedback impedance. Depending on the nature of z_1 and z_2 Expression (11) becomes:

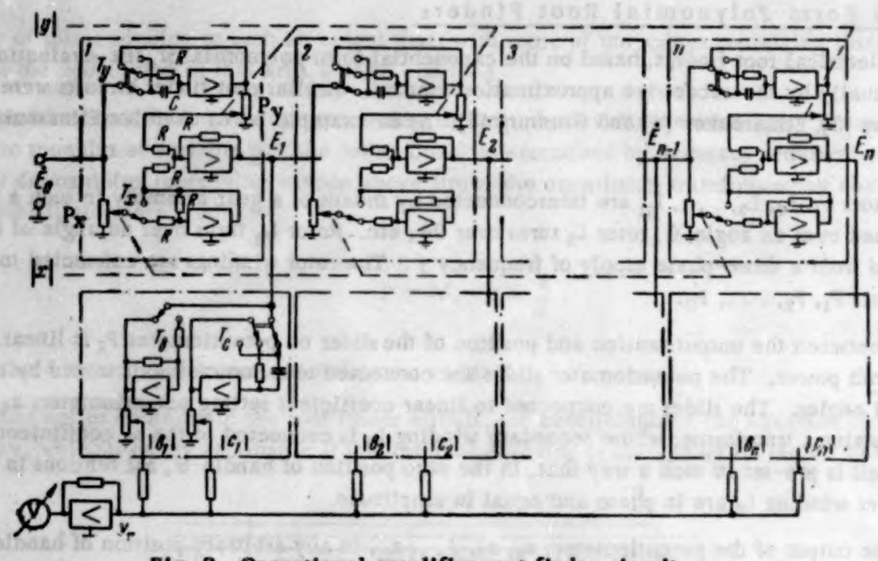

Fig. 2. Operational amplifier root finder circuit.

a) with $z_1 = z_2 = R$

$$E_1 = -E_0; \qquad (12)$$

b) with $z_1 = \frac{1}{j2\pi fC}$, $z_2 = R$, $\frac{1}{2\pi fC} = R$

$$E_1 = -jE_0; \qquad (13)$$

c) with $z_1 = R$, $z_2 = \frac{1}{j2\pi fC}$, $R = \frac{1}{2\pi fC}$

$$E_1 = jE_0. \qquad (14)$$

In addition to above operations the amplifier can also register a sum. Utilizing these properties of operational amplifiers Bubb designed a simple circuit which multiplied ac tensions by complex quantities p = x + jy. It is easy to show that tension E_1 at the output of element 1 (Fig. 2) is related to tension $\dot{E}_0 = |E_0| e^{j2\pi ft}$ at the input by the expression

$$E_1 = (x + jy) E_0 = pE_0. \qquad (15)$$

The values of $|x|$ and $|y|$ are set on linear potentiometers P_x and P_y and their signs are determined by the position of tumbler switches T_x and T_y.

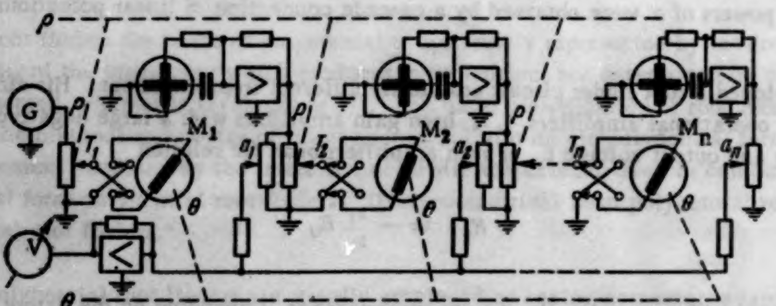

Fig. 3. Operational amplifier and magslip root finder circuit.

If several such circuit elements are connected in series then the tension at the output of the first element will be $\dot{E}_1 = pE_0$, of the second $-\dot{E}_2 = p^2E_0$, the third $\dot{E}_3 = p^3E_0$, etc. All the slides P_x are connected to a common axis $|x|$, and the slides P_y to axis $|y|$. The corresponding tumbler switches must also be mechanically interconnected. A similar circuit element is used for multiplying the obtained powers of p^ν by complex coefficients $a_\nu = b_\nu + jc_\nu$. Next the terms $a_\nu p^\nu E_0$ are added up on the operation amplifier Y_Σ, whose output tension is measured in instrument V. The roots are evaluated manually by the successive approximations method. This circuit formed the basis of Lukashevich's root finder [8].

A disadvantage of setting up variable $\underline{p}$ in Cartesian coordinates is the switching required to pass from one quadrant to another. This disadvantage is eliminated if polar coordinates are used for the solution of the problem. Such a changeover is easily carried out [9] by replacing the potentiometers P_x and P_y by magslips M as it is shown in Fig. 3.

The magslip cosine winding is connected to the operational amplifier through a resistance, whereas the sine winding is connected through a capacity; thus the circuit consisting of a modulus ρ setting potentiometer, a magslip and one operational amplifier provide for the multiplication of the input tension by $p = \rho(\cos\theta + j\sin\theta)$, in which θ can vary between 0 and 360°. The switch T serves to reverse the sign of the tension.

III. Trigonometrical Form Polynomial Root Finder

Basic operations which must be performed by a trigonometrical form polynomial root finder when setting up terms $a_\nu\rho^\nu\cos\nu\theta$ and $a_\nu\rho^\nu\sin\nu\theta$ of Expression (6) are: obtaining powers of ρ, taking into account trigonometrical relations of the type of $\cos\nu\theta$ and $\sin\nu\theta$ and multiplication. For multiplication electromechanical devices are mainly used (potentiometers, variacs, magslips, etc.) in which one of the factors is the electrical tension and the other mechanical displacement. Let us call multiplicand the factor provided electrically and multiplier that provided mechanically. Depending on which function serves as the multiplier the trigonometrical form polynomial root finders can be divided into three groups: in group I ρ^ν serves as the multiplier, in group II it is $\cos\nu\theta$ and group III includes root finders with a purely electronic method of multiplication.

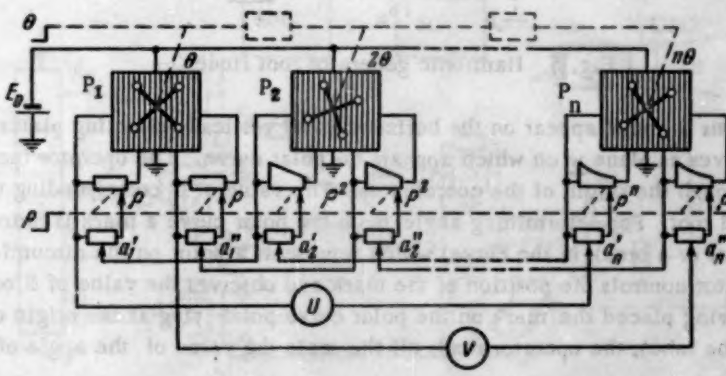

Fig. 4. Sine-cosine potentiometer root finders.

Group I circuit. The first circuit of a trigonometrical form polynomial root finder was proposed by Elizarenkov [4] (Fig. 4). It consists of sine-cosine potentiometers P_1 to P_n whose slides are connected by gear assemblies in such a way that by turning with handle θ the first potentiometer over an angle θ, the second is made to rotate over an angle 2θ, the third over 3θ, etc. The inputs of the potentiometers are connected to a common dc tension E_0. The sine and cosine outputs of potentiometer P_1 are connected to two similar modulus setting potentiometers with a linear winding, P_2 is connected to potentiometers with a quadratic winding, P_3 to potentiometers with a cubic winding, etc. The slides of all these potentiometers are connected to a common axis for setting ρ, and the potentiometers themselves to the linear coefficient setting potentiometers a_1' and a_1'', a_2' and a_2'', a_3' and a_3'', etc. At the series connected outputs of a_1', a_2', a_3', . . . and a_1'', a_2'', a_3'', . . . two respective voltages are formed $U = \sum_{\nu=1}^{n} a_\nu\rho^\nu\cos\nu\theta$ and $V = \sum_{\nu=1}^{n} a_\nu\rho^\nu\sin\nu\theta$ which are measured on instruments. The evaluation of roots is made manually by the successive approximations method.

Root finding in this circuit was made automatic by G. M. Zhdanov [10] who proposed to operate the ρ-axis by means of a servo-unit whose input is connected to tension U and to connect tension V to the input of a servo-unit controlling the θ-axis. The servo-units stop when tensions U and V become zero, i. e., when ρ and θ become coordinates of the required root. As it has already been pointed out such a root finder becomes a closed automatically controlled system. It would appear that the stability of such a system and the time taken to achieve balance will depend on the character of coefficients in the original equation. No experimental data for the operation of such a root finder exists, since as far as it is known the instrument was never constructed.

If axis θ in the circuit of Fig. 4 is rotated at a constant speed roots can be found by the inspection method. This rotation will also make harmonic frequencies appear at the outputs of the sine-cosine potentiometers. It is, obviously, better to generate such frequencies in a purely electronic manner by means of a harmonic oscillator, which was in fact carried out by Tischner [22] and Bader [11] (Fig. 5). The formation of harmonics in this case is carried out by means of generator G, a frequency multiplying circuit FMC and filters F_1 to F_n.

The use of two modulus and two coefficient potentiometers in each channel is a disadvantage of the sine-cosine potentiometer root finders. In the harmonic generator circuit each channel has only one such potentiometer and the formation of the sine and cosine components of Expression (6) is achieved by phase splitting elements PS.

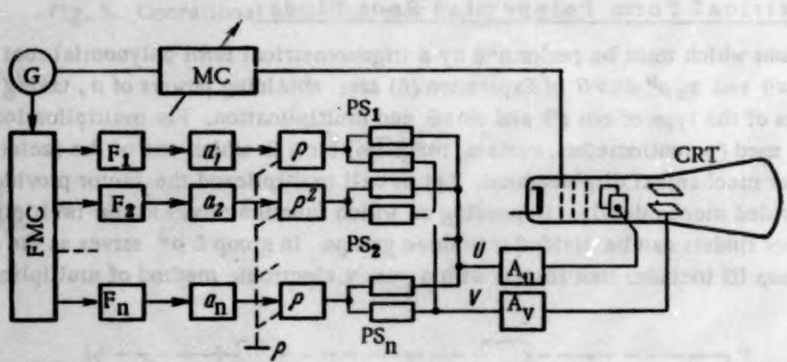

Fig. 5. Harmonic generator root finder.

The aggregate tensions U and V appear on the horizontal and vertical deflecting plates of a cathode-ray tube CRT, whose screen serves as plane w on which appears the polar curve. The operator turns handle ρ striving to make the curve pass through the origin of the coordinates. The value of ρ corresponding to this position is the modulus of the required root. For determining angle θ on the polar curve a mark is formed electronically (in the shape of a bright spot or a break in the curve) which represents a point on the circumference of radius ρ on the plane w. The operator controls the position of the mark and observes the value of θ on a scale of the measuring circuit MC. Having placed the mark on the polar curve point lying at the origin of the coordinates on plane w (the screen of the tube), the operator reads off the scale the value of the angle of the root.

Choudhury [12] proposed a root finder circuit using an artificial long line. As is known tension $\dot{E}$ at a distance χ away from the end of such a line is equal to

$$\dot{E} = \dot{E}_k \operatorname{ch} \gamma\chi + \dot{I}_k z \operatorname{sh} \gamma\chi,^* \tag{16}$$

where E_k and I_k are the tension and current, respectively, at the end of the line, z is the characteristic impedance and $\gamma = \beta + j\alpha$ is the propagation constant. In a line without losses $\beta = 0$ and Eq. (16) becomes

$$\dot{E} = \dot{E}_k \cos \alpha\chi + j\dot{I}_a z \sin \alpha\chi. \tag{17}$$

In the case of free running with open circuited output terminals we have

$$\dot{E} = \dot{E}_k \cos \alpha\chi. \tag{18}$$

* 'ch' ≡ 'cosh', 'sh' ≡ 'sinh' – Publisher.

With short circuited terminals we obtain

$$\dot{E} = j\dot{I}_k z \sin \alpha\chi. \tag{19}$$

If a long line consists of nl similar low pass filter sections, χ denotes the number of sections between the end of the line and a given point. For a point νl sections away from the end of the line, expressions (18) and (19) will become

$$\dot{E}_\nu = \dot{E}_k \cos \nu\alpha l, \quad \dot{E}_\nu = j\dot{I}_k z \sin \nu\alpha l \tag{20}$$

or if one assumes $\alpha l = \theta$:

$$\dot{E}_\nu = \dot{E}_k \cos \nu\theta, \quad \dot{E}_\nu = j\dot{I}_k z \sin \nu\theta. \tag{21}$$

In the passband of low-pass constant-k type filters* $\alpha = 2 \text{ arc sin } \dfrac{f}{f_c}$, where f_c is the cutoff frequency. Hence,

$$\theta = 2l \text{ arc sin } \frac{f}{f_c}. \tag{22}$$

In Choudhury's root finder (Fig. 6) the artificial delay line DL is connected to the swinging frequency generator SFG, controlled by the sawtooth oscilloscope time-base generator SG. The limits of frequency f variations are selected in such a way that $0.383 \leq f/f_c \leq 0.707$, and l is taken as 8. θ varies between 0 and 360°.

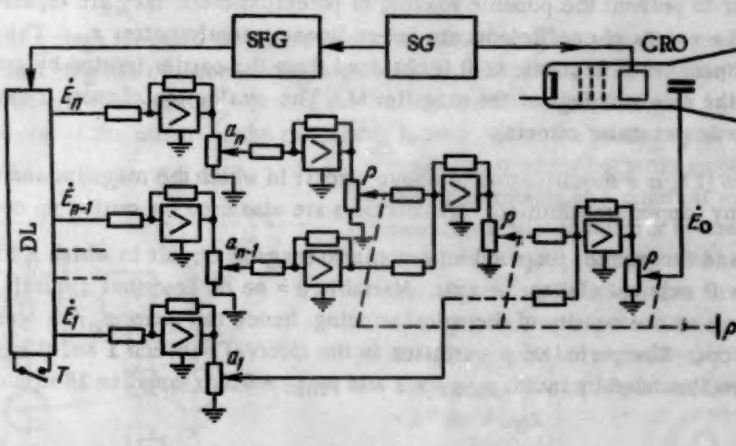

Fig. 6. Delay line root finder circuit.

Tension $\dot{E}_\nu$ is taken from the output of the required filter section, connected through a separating amplifier to the coefficient setting potentiometer a_ν and then multiplied by ρ^ν and simultaneously added to other tensions. In this circuit, as distinct from the circuits described above, it was possible to use linear potentiometers for multiplying by various powers of ρ. The output tension E_0 corresponds either to component V or U of Expression (6) if the output terminals of the line are closed or open. This tension is impressed on the vertical plates of the oscilloscope whose horizontal time-base controls the frequency f.

* Filters with their series and shunt arms consisting of two-terminal networks whose product is a constant are known as "constant-k" type filters.

A rough evaluation of roots is supposed to be done manually by the inspection method alternately shorting and opening the line output terminals* with the switch T. An accurate value of coordinates is obtained by the successive approximations method.

Group II circuits. The magslip circuit [14-16] is being widely used (Fig. 7). Magslips MG permit one to vary rapidly and thus change over from the successive approximations method [14] to the manually operated inspection method [15, 16]. Continuous variation of coordinate θ is achieved, in this case, by means of a motor M which turns the magslip rotors through a gear assembly. The use of magslips made it possible to change the circuit over to ac working.

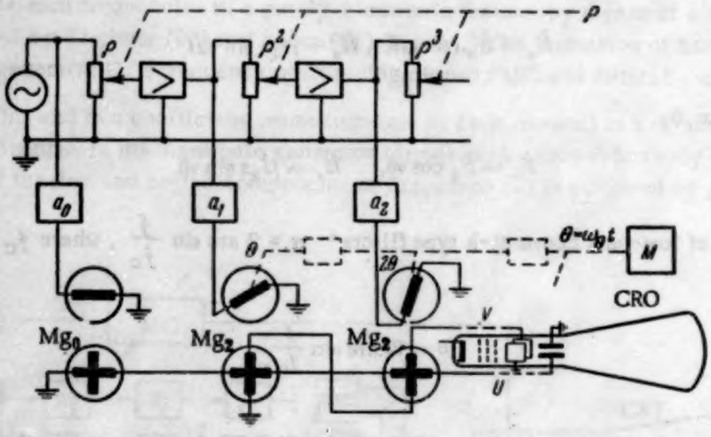

Fig. 7. Magslip root finder circuit.

Various powers of ρ are obtained by means of a cascade connection of linear potentiometers with a common slider axis. In order to prevent the possible loading of potentiometers, they are separated by amplifier with a gain equal to one. The values of coefficients are set on linear potentiometers a_ν. The formation of the real U and imaginary V components of Expression (6) is obtained from the carrier tension by grouping in a series connection the cosine and the sine windings of the magslips M. The evaluation of roots is carried out in the same manner as in the harmonic generator circuit.

Adler's instrument [17] is a modification of above circuit in which the magslips and modulus setting potentiometers are replaced by tapped transformers; transformers are also used for setting up coefficients.

Calvert,Johnston and Singer [18] proposed an automatic magslip circuit in which a motor drives with constant speed not only the θ axis but also the ρ axis. Variable $p = \rho e^{j\theta}$ describes a spiral $\rho = (\omega_\rho / \omega_\theta)\theta$. The working accuracy depends on the density of the spiral winding, hence the ratio $\omega_\rho / \omega_\theta$ was taken as 0.01, and the frequency f as 400 cps. The period of ρ variation in the interval between 1 and 0.1, i. e., the inspection time of plane p ring area, bounded by radius $\rho_{max} = 1$ and $\rho_{min} = 0.1$ is equal to 18 minutes.

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* The method of alternately shorting and opening the end of the line cannot be considered convenient. In order to generate U and V simultaneously two similar delay lines are required, which is not expedient. It is possible, however, to indicate a delay line root finder design free from above disadvantage.

If the line is terminated in its characteristic impedance and it is noted that $\dot{I}_k = \dot{E}_k / z$ it becomes possible possible to write Expression (17) in the form

$$\dot{E}_\nu = \dot{E}_k \cos \alpha\chi + j \frac{\dot{E}_k}{z} z \sin \alpha\chi = \dot{E}_k e^{j\alpha\chi} = \dot{E}_k e^{j\nu\alpha l} = \dot{E}_k e^{j\theta\nu} \tag{23}$$

If $\dot{E}_\nu$ is multiplied by a_ν and ρ^ν and added to the remaining tensions, the tension E_0 obtained at the output of the circuit will represent the entire polynomial w in the form (5) and not a part of it only as is the case in Choudhury's circuit. The circuit thus obtained should, of course, be classified as an exponential form polynomial root finder.

The demodulated tensions U and V are impressed on an electronic zero indicator which clamps the zero values of $|U|$ and $|V|$, stops the motor and informs the operator by a signal.

An accurate evaluation of the root thus obtained is made manually by the method of successive approximations. In this root finder certain measures have been taken to improve accuracy. For instance, the gain of the separating amplifiers changes automatically according to the value of ρ, helical potentiometers are used for setting moduli.

The main object of making root finding automatic is not so much for the purpose of saving time in solving equations as for obtaining (usually on a cathode-ray tube screen) a picture of root distribution which would facilitate the observation of root migration when any coefficient in the equation is changed. For this purpose the inspection process of the complex plane selected area must be repeated periodically, and the frequency of repetition Ω must be sufficiently high in order to make it possible to obtain root trajectories in the form of curves on the tube screen.

The above magslip automatic circuit does not permit one to obtain root trajectories owing to the very low repetition frequency ($\Omega = \frac{\omega_\rho}{2\pi} = 10^{-3}$ cps), moreover, the time spent on root finding is probably increased instead of being decreased as compared with non-automatic circuits [15-17]. The root finding can only be speeded up in this circuit by increasing the carrier frequency f, limited by the magslip operating supply frequency, which at the present time cannot exceed 1000 cps. Another drawback of all carrier automatic root finder circuits is the necessity of demodulating tensions U and V before they can be impressed on the zero indicator. In order not to distort the envelope in demodulation, the repetition frequency Ω has to be kept low. The use of a carrier frequency, therefore, in automatic root finder circuits must be considered inexpedient.

A root finder without a carrier on the basis of a manually operated inspection method was constructed by Marshall [19] (Fig. 8). The powers of ρ are formed in it in the same manner as in the magslip circuit, the two differ, however, in the fact that the former has its potentiometer cascade fed by dc. Amplitude modulated square-wave pulses of a frequency $\nu \frac{\omega_\theta}{2\pi}$ are obtained from tension $E_0\rho^\nu$ by means of an electromechanical modulator. The modulator contains n rings mounted on a common axis; the rings have equally spaced conducting and insulated segments. The first ring has one conducting segment, the second has two, the third three, etc. The conducting segments are connected to a common point of the circuit, so that when the brush is in contact with one of these segments the tension at the output of the modulator is zero, when the brush comes over an insulated segment the tensions becomes equal to $E_0\rho^\nu$. When the motor M rotates at a constant speed the ν-th channel modulator will produce a square-wave tension whose amplitude will be proportional to ρ^ν. Filter F_ν picks out of these oscillations the first harmonic tension which is multiplied on a potentiometer by coefficient a_ν and then impressed on a phase-splitting RC network whose output tensions are equal to $E_0 a_\nu \rho^\nu \cos \nu\omega_\theta t$ and $E_0 a_\nu \cdot \rho^\nu \sin \nu\omega_\theta t$.

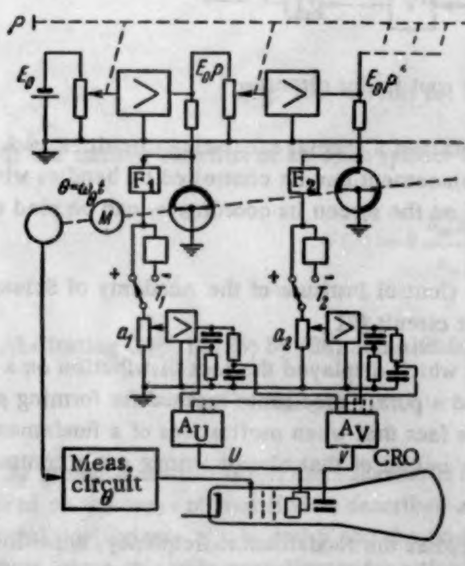

Fig. 8. Square-wave modulator root finder circuit.

The signs of the coefficients are set by the switch T. The aggregate tensions U and V form on the screen of the cathode-ray tube a polar curve. The quantity $\frac{\omega_\theta}{2\pi}$ is made to equal 60 cps.

Lofgren [20] substituted the electromechanical modulator by an electronic one, this allowed him to raise ω_θ and change over to an automatic method of root finding. Automatic variations of ρ can be produced without any moving parts by utilizing the exponential relationship of ρ to time. Such a law of variation of ρ can be relatively easily produced by periodic switching

of RC circuits. As is known the raising to power ν of an exponential quantity will produce another such quantity but with a time constant $1/\nu$ times the former value. If the time constant of the RC networks τ = RC is selected in such a way that among the various channel time constants the following relation holds:

$$\tau_1 = 2\tau_2 = 3\tau_3 = \ldots = \nu\tau_\nu = \ldots n\tau_n, \tag{24}$$

the tensions produced by the networks of the second, third, etc. channel will be respectively the square, cube, etc. of the first channel RC network tension. The switching frequency in Lofgren's root finder is taken as $\Omega = \omega_\rho/2\pi =$ $= 1$ cps and the value of τ_1 is selected to make ρ vary in the limits $^1/_2 \leq \rho \leq 1$.

The square-wave harmonic frequency oscillations are generated by a special frequency dividing circuit. The quantity $\frac{\omega_\theta}{2\pi}$ is approximately equal to 1000 cps. The formation of the sine and cosine components of Expression (6) is achieved without phase-splitting networks by an operational amplifier-summator and summator-integrator.

The picture of root distribution of the required equation is produced in the form of light spots on a cathode-ray tube screen with a long afterglow. The screen should simulate part of the complex root screen. For this purpose the tube deflecting systems are supplied with tensions which displace the ray along the same spiral $\rho =$ $= e^{-\theta/\omega_\theta\tau_1}$, as the one controlling the automatic variation of ρ in the counting part of the instrument. Normally the ray is suppressed and it is only switched on for the instant when the tensions U and V, which represent the real and imaginary components of the polynomial, simultaneously become zero when they are clamped by the zero indicator.

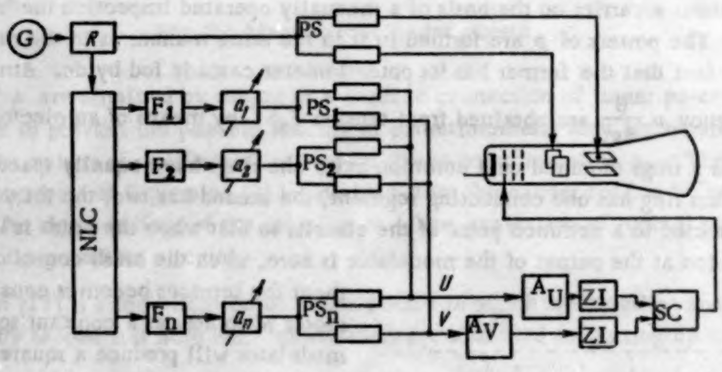

Fig. 9. Electronic multiplier root finder circuit.

For an accurate evaluation of the root coordinates thus obtained a special measuring circuit is used. This circuit produces on the tube screen a movable mark, whose displacement can be controlled by handles with scales. When the mark is made to coincide with a root which appeared on the screen its coordinates can be read off the scales.

The automatic root finder of the Automation and Remote Control Institute of the Academy of Sciences, USSR was constructed on the basis of the square-wave modulator circuit [21].

Group III circuits. The first automatic root finder circuit which displayed the root distribution on a screen of a cathode-ray tube was developed by Rasch [22] who proposed a purely electronic method for forming $\rho^\nu \cos \nu \cdot \omega_\theta t$ and $\rho^\nu \sin \nu\omega_\theta t$ components. This method is based on the fact that when oscillations of a fundamental frequency $\rho \cos \omega_\theta t$ are impressed on a nonlinear circuit, at the output of that circuit among other components will also appear tensions $\rho^\nu \cos \nu\omega_\theta t$ which can be filtered out.

Rasch's circuit (Fig. 9) consists of a generator G which supplies the fundamental frequency, adjusting unit A, for adjusting the amplitude of ρ, and the above-mentioned circuit with a nonlinear characteristic NLC which serves simultaneously for multiplying and raising to the power. This circuit is followed by filters F_1 to F_n which pick out the required frequencies and by units for setting coefficients a_ν. The separation of tensions into their sine and cosine components is achieved by means of phase splitters PS.

The picture of root distribution is obtained in the same way as in the automation square-wave modulated root finder described above.

On the basis of this circuit Glubrecht [22] constructed an automatic inspection method root finder with an alternative manual operation; for the latter purpose a screen representing plane w was incorporated in the instrument in addition to the screen simulating plane p. The change over to manual operation is intended for accurate evaluation of root coordinates. For generating oscillations of the form $\rho^{\nu} \cos \nu \omega_{\theta} t$ instead of a nonlinear circuit an electron tube is provided whose control grids have tensions $\rho \cos \omega_{\theta} t$ and $\rho^{\nu-1} \cos(\nu-1)\omega_{\theta} t$ impressed on them. The output tension contains component $\rho^{\nu} \cos \nu\omega_{\theta} t$ which is filtered out.

IV. Root Finders for Solving ACS Characteristic Equations

An automatic control system (ACS) characteristic equation can be given in the form

$$G(p) + 1 = 0. \tag{25}$$

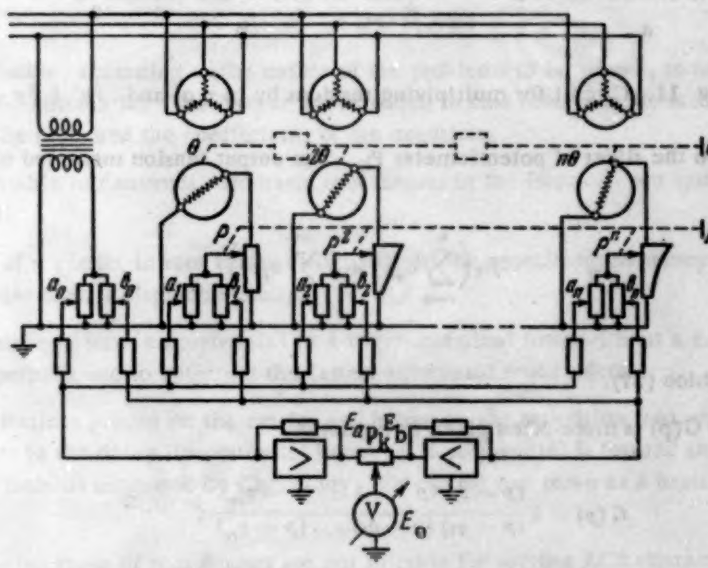

Fig. 10. Circuit for solving ACS characteristic equations.

If the transfer function of an open system G(p) is given by the expression

$$G(p) = k \frac{b_m p^m + b_{m-1} p^{m-1} + \ldots + b_0}{a_n p^n + a_{n-1} p^{n-1} + \ldots + a_0}, \tag{26}$$

after substituting G(p) in (25) by (26) and eliminating the common denominator we obtain

$$a_n p^n + a_{n-1} p^{n-1} + \ldots + a_0 + k(b_m p^m + b_{m-1} p^{m-1} + \ldots + b_0) = 0. \tag{27}$$

By grouping similar terms the equation is reduced to the usual (1) form of an algebraic equation which can be solved on the types of root finders described above. If, however, the object is to investigate the relation between the initial coefficients a_{ν}, b_1 and k and the position of roots, the reduction of the equation to Form (1) is not expedient, since this only complicates investigation. It is desirable to be able to solve the characteristic equation in the form (27).

It is easy to construct devices for the solution of such equations on the basis of root finders of any type. For this purpose it is necessary to connect in parallel with the a_ν potentiometers others for setting the b_i coefficients and to connect them to a circuit similar to the one to which the a_ν potentiometers are connected.

As an example of such a device a circuit based on a selsyn root finder is given in Fig. 10 (secondary details have been omitted). Tensions E_a and E_b at the outputs of the summing amplifiers can be written down as

$$E_a = E_0 \sum_{\nu=0}^{n} a_\nu \rho^\nu e^{j\nu\theta} = E_0 \sum_{\nu=0}^{n} a_\nu p^\nu, \quad E_b = E_0 \sum_{i=0}^{m} b_i \rho^i e^{ji\theta} = E_0 \sum_{i=0}^{m} b_i p^i. \tag{28}$$

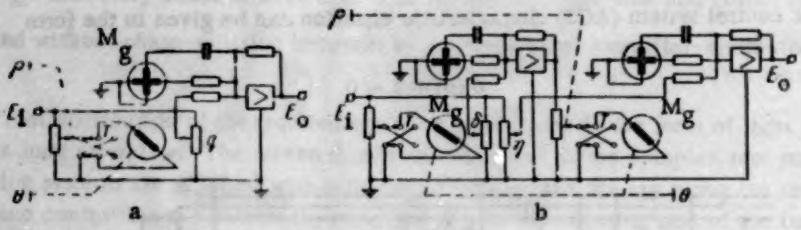

Fig. 11. Circuit for multiplying tensions by $(p-q)$ and $(p^2+\delta p+\eta)$

Coefficient k is set on the slider of potentiometer P_k. The output tension measured on meter V is equal to

$$E_0 = AE_0\left(\sum_{\nu=0}^{n} a_\nu p^\nu + k\sum_{i=0}^{m} b_i p^i\right), \tag{29}$$

which corresponds to Expression (27).

The transfer function G(p) is more often given in the form

$$G(p) = k\frac{(p-z_1)(p-z_2)\dots(p-z_m)}{(p-q_1)(p-q_2)\dots(p-q_n)}, \tag{30}$$

where $z_1, z_2, \dots, z_m$ are zeros and $q_1, q_2, \dots, q_n$ are poles. Usually one is interested in the relations between the zeros and poles, and the position of roots of the characteristic equation, therefore, the transformation of Expression (30) to the form (26) by the opening of brackets is not admissible.

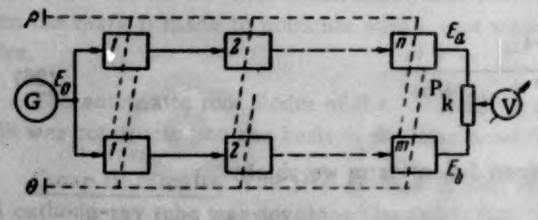

Fig. 12. Block schematic of a root finder for the solution of equations

$$(p-q_1)(p-q_2)\dots(p-q_n)+k(p-z_1)(p-z_2)\dots(p-z_m)=0.$$

Tensions E_a and E_b:

$$E_a = E_0(p-q_1)(p-q_2)\dots(p-q_n), \quad E_b = E_0(p-z_1)(p-z_2)\dots(p-z_m).$$

The construction of a root finder for the solution of ASC characteristic equations when G(p) is given in the form (30) is a more complicated problem mainly because products of complex numbers have to be formed in the counting part of the instrument. Nevertheless, by taking as the basis a root finder with operational amplifiers and magslips it is possible to make up a circuit which would evaluate roots of such equations.

The substitution of G(p) in (25) by (30) and the elimination of the common denominator will give

$$(p-q_1)(p-q_2)\dots(p-q_n)+ k(p-z_1)(p-z_2)\dots(p-z_m)=0. \tag{31}$$

The poles q_ν and zeros z_l can be both real and complex conjugate numbers. Hence, the circuit must contain components capable of multiplying tensions by complex expressions of the type of $(p - q)$ and $(p^2 + \delta p + \eta)$ where $\underline{q}$, δ and η are real numbers. For these operations circuits shown in Fig. 11 can be used. The output tension in the circuit of Fig. 11a is

$$E_0 = (p - q)E_1. \tag{32}$$

The circuit in Fig. 11b reproduces the relationship

$$E_0 = (p^2 + \delta p + \eta)E_1. \tag{33}$$

The signs of coefficients $\underline{q}$, δ and η are produced by switch T.

A circuit for producing the left-hand side of Eq. (31) and consisting of such components is shown in Fig. 12. Amplification factor $\underline{k}$ is set by means of potentiometer P_k. A root finder constructed on the basis of this circuit can be made to evaluate roots either manually or automatically.

SUMMARY

1. It is advisable, according to the nature of the problems to be solved, to have two types of root finders: semi-automatic and automatic. The former are intended to find roots and the latter mainly to investigate the relation between the roots and the coefficients of the equations.

2. It is advisable to construct automatic root finders in the form of open systems which find roots by the inspection method.

3. The use of a carrier in root finder circuits limits the repetition frequency and thus makes it difficult to obtain trajectories of root displacements.

4. It is possible to form a polynomial in a trigonometrical form without a carrier, hence, the use of this polynomial form permits one to construct the fastest automatic root finders.

5. The limitations placed on the carrier and hence on the repetition frequency by the use of selsyns and magslips are absent in the delay line circuit. Hence, if a polynomial is formed in an exponential form (and not a trigonometrical form as suggested by Choudhury) this circuit can serve as a basis for the construction of a fast root finder.

6. The existing types of root finders are not suitable for solving ACS characteristic equations, it is, therefore, desirable to construct devices which would investigate the relations not only between the roots of a closed ACS equation, and the poles, but also the zeros of an open system transfer equation. Possible circuits for such devices have been suggested in part IV of the article.

7. By utilizing the principle of continuous inspection (scanning) of a complex plane it is possible to construct, on the basis of existing root finders, a device for conformal mapping.

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CHRONICLE

NATIONAL CONFERENCE AT BUCHAREST ON THE QUESTION OF AUTOMATION OF PRODUCTION

During June 5-8, 1957, the Second National Conference on the Question of Automation of Production was held in Bucharest (Rumanian National Republic). Invited to participate in the Conference were scientists and specialists in automation from Bulgaria, Poland, Hungary, Rumania, Czechoslovakia, the USSR and Yugoslavia.

In the majority of cases, these were contributors to academic, research and professional institutes who simultaneously engaged in pedagogical work. Among them were: Corr. Mem. Bulgarian Acad. Sci. A. Bolevskii, Prof. D. Mitrovich (Yugoslavia), Dr. I. Benesh (Czechoslovakia), Prof. Libson (Poland) and others.

A total of 99 papers were presented at the Conference, including 91 from Rumania, 2 from Czechoslovakia, 2 from Hungary, 1 from Poland, 2 from the USSR and 1 from Yugoslavia.

The delegation from the USSR included two contributors to the IAT AN SSSR, Prof. N. N. Shumilovskii and Eng. I. M. Makarov.

Besides participating in the work of the Conference, our delegates had the tasks of familiarizing themselves with the arrangements for research work in automation and related fields in the scientific institutions of the AN RNR and in the professional institutes, familiarizing themselves with the state of automatic production in the factories and, in addition, establishing scientific contacts with the scientific institutions and the scientists of the RNR and of the other countries represented at the Conference.

The National Conference on Automation was called at the initiative of the Academy of Science of the RNR. The Academy does not have an institute devoted to the questions of automation, but there is a committee on automation, headed by Academician I. S. Georgiu. The Committee has two scientist-secretaries, contributors to academic and professional institutes, to institutions of higher learning, industrial plants, ministries and Gosplan RNR. All the organizational work involved in summoning the Conference was done under the aegis of this Committee.

Several months before the opening of the Conference, the organizing committee sent requests to the desired foreign participants requesting that texts of papers be furnished by the proper dates.

The Conference opened on June 5 in one of the buildings of the Bucharest State University. Presiding Academician I. S. Georgiu opened the session with a few words welcoming the guests and participants in the Conference and outlining the problem which the Conference was to consider. Further words were added by Prof. N. N. Shumilovskii, welcoming the participants in the name of the Institute for Automation and Remote Control of the Academy of Sciences of the USSR, and wishing success to the Conference in its work. Short welcoming addresses were made by representatives of the other countries, after which the order of business of the Conference was announced.

The participants were given programs containing the theses of the Rumanian delegates' papers, these programs being printed in the Rumanian language, which somewhat attenuated the intensity of the discussions. At the plenary session, papers were presented by guest delegates Prof. N. N. Shumilovskii, Dr. I. Benesh and Prof. D. Mitrovich, dealing with the state of automation and measurement technology in their countries in various domains of industry. Then followed Rumanian delegates: Academician G. K. Moisil, Prof. K. Penesku, Engr. D. Damsker, Engr. M. Maresh, Corr. Mem. RAN A. Avramensku and others, who, in their papers, dwelt on the questions of automating various branches of industry in the RNR, on questions of terminology in automata, and also on questions of preparing personnel for the automation of industry.

Further work of the Conference was carried out in three sections. The first section dealt with the theoretical basis of automation. The second section considered the general questions of automation, measurement and control of productive processes, and the third section was concerned with questions of automation, measurement and technological control in various branches of industry.

The first section considered questions of linear and nonlinear theories of automatic control, the theory of relay-contact designs, statistical methods of investigating automatic control systems, and certain questions in the theory of electric drives.

Among the papers presented at this section, we mention the following: "Synchronous Motors and Motors with Variable Inductance as Servomechanisms with Proportional Control" by Corr. Mem. AN RNR M. Marinesku and Engr. G. Iankylesku, "A Contribution to the Simplification of the Stability Conditions of A. I. Lur'e" by Engr. V. Popov, "Construction of the Transient Response in Systems with Nonlinear Regulators" by Engr. D. Damsker, "Active Correcting Links" by Engr. K. Vazak, "A Method of Simplifying the Analysis of Continuous Linear Systems of Automatic Control" by K. Penesku, "A Method of Designing Discontinuous Automatic Control Systems" by Engr. N. Shtefanesku and "The Characteristic Equation of a Tripping Relay" by Academician G. Moisil of the AN RNR.

Papers presented in the first section by foreign delegates included: "Some Questions in the Theory of Nonlinear Automatic Control Systems" by Dr. S. Vengzhin (Poland), "Statistical Method for Investigating the Dynamics of Control Systems" by Dr. I. Benesh (Czechoslovakia) and "New Method of Determining the Characteristics of Single-Phase Electric Spindles" by Engr. L. Ianoki (Hungary).

In the papers of the second section, in addition to questions of automation, measurement and control of individual productive processes, there were considered the questions of simulating, creation of digital computers and also methods for the design and construction of individual elements of automatic devices. Together with these, papers were presented dealing with the economic questions of automation. A total of 27 papers were given at this section, including: "Economic Considerations in Automation in Capitalist Countries" by Engr. E. Balash, "Automation in Countries Within the Socialist Camp" by I. Teodorov and I. Lemin, "The Electronic Computer of the Institute for Nuclear Physics of the AN RNR" by Engr. V. Tom, "Electrical Measurement of the Moisture Content of Materials" by Engr. M. Steru, "On the Planned Radio-Tele-Control Systems to be Applied to Objects Moving at High Speeds" by Engr. R. Konstantinesku and "Questions as to the Method of Designing Determining Amplifiers for Electronic Models" by Engr. S. Shekhter. Engr. I. M. Makarov (USSR) gave a paper on the subject, "Choosing Optimal Electrodynamic Couplings for Automatic Control Systems."

The papers of the third section presented the practical results of the work carried out in the RNR in the domain of automation of different branches of industry: petroleum, chemical, light industry, textiles, food, printing, and others. Some attention was also given to work in the automation of communication. That more than 30 papers on the automation of the national economy of the RNR were given bespeaks the fact that, in Rumania, a sufficiently large group of specialists in the domain of automation has been assembled. The plenary session of the Conference was held on June 8. Both scientists representing the ministeries and Gosplan of the RNR and guests took part in the discussions of this session. In his concluding remarks, Academician I. S. Georgiu mentioned the increased interest in Rumania in the problems of automation and the marked rise in the general level of research and planning of automatic devices, which argues well for the further development of automata and measuring technology in the Rumanian National Republic.

In the resolutions of the Conference, together with important organizational and practical measures bearing directly on the questions of the development of the theory and practice of automation in the RNR, particular attention was given to the necessity for increasing scientific and technical cooperation with other countries and, as the first step, with the countries in the socialist bloc.

To this end, it was proposed to develop the interchange of scientific and technical documentation, to work out a program of national conferences on question of automation with participation of the other national democratic countries, to organize exchange visits of specialists from Rumania and from foreign countries, in particular from the national democratic countries, to take steps to include Rumania in the recently created International Federation for Automatic Control.

Not only in the Conference resolutions, but also in personal conversations with the representatives of a number of countries, there were expressed hopes for the further strengthening of scientific ties, and of organizing a single group of specialists in the domains of automation, remote control and measuring technology. It was proposed to summon a conference to consider the results of the most important scientific and practical work carried out in the countries of the democratic bloc, to interchange regularly printed work so that they might be periodically published in these countries, to exchange attempts to apply certain methodologies in scientific investigations, and also to organize joint scientific work in the domain of automata.

The delegates to the Conference had the opportunity to visit the Institute for Nuclear Physics, located not far from Bucharest, where a working model of an electronic digital computer was demonstrated.

On June 9, after the close of the Conference, guests and members of the organizing committee had an excursion to a newly constructed, well-designed petroleum refinery in Ploesti.

On returning from the excursion, the guests were taken in hand by the scientist-secretaries of the Presidium of the AN RNR and held conversations with the leading scientists of the AN RNR.

In addition to the organizations to which visits were provided for in the Conference program, the Soviet delegates visited the Energetics Institute of the AN RNR, the Electro-Technology Institute and the Military-Technical Academy in Bucharest, and also a chemical factory producing superphosphates and sulfuric acid.

In conclusion, it is desirable to mention that, both on the part of the organizing committee and on the part of all its members, the delegation from the USSR experienced the burgeoning of friendly, warm relationships.

I. M. Makarov and N. N. Shumilovskii

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